MHT CET 2023 : 14th May Morning Shift

Mathematics

Question 1

The negation of the statement

"The number is an odd number if and only if it is divisible by 3."

Options:

A. The number is an odd number but not divisible by 3 or the number is divisible by 3 but not odd.

B. The number is not an odd number iff it is not divisible by 3.

C. The number is not an odd number but it is divisible by 3.

D. The number is not an odd number or is not divisible by 3 but the number is divisible by 3 or odd.

Answer: A

Solution:

Let p : The number is an odd number,

q : The number is divisible by 3

$\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

$\therefore$ The negation of the given statement is 'The number is an odd number but not divisible by 3 or the number is divisible by 3 but not odd'.

Question 2

Two cards are drawn successively with replacement from well shuffled pack of 52 cards, then the probability distribution of number of queens is

Options:

A.

$X = x$	0	1	2
$P[X = x]$	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

B.

$X = x$	0	1	2
$P[X = x]$	$\frac{1}{169}$	$\frac{24}{169}$	$\frac{144}{169}$

C.

$X = x$	0	1	2
$P[X = x]$	$\frac{24}{169}$	$\frac{1}{169}$	$\frac{144}{169}$

D.

$X = x$	0	1	2
$P[X = x]$	$\frac{1}{169}$	$\frac{25}{169}$	$\frac{143}{169}$

Answer: A

Solution:

Let X denote the number of queens.

$\therefore$ Possible values of X are $0, 1, 2$.

$$P(\text{ queen }) = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{ not a queen }) = \frac{48}{52} = \frac{12}{13}$$

$$P(X = 0) = \frac{12}{13} \times \frac{12}{13}$$

$$= \frac{144}{169}$$

$$P(X = 1) = \left(\frac{1}{13} \times \frac{12}{13}\right) + \left(\frac{12}{13} \times \frac{1}{13}\right)$$

$$= \frac{12}{169} + \frac{12}{169} = \frac{24}{169}$$

$$P(X = 2) = \frac{1}{13} \times \frac{1}{13}$$

$$= \frac{1}{169}$$

The probability distribution of X is

$X = x$	0	1	2
$P[X = x]$	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Question 3

For an initial screening of an entrance exam, a candidate is given fifty problems to solve. If the probability that the candidate can solve any problem is $\frac{4}{5}$, then the probability, that he is unable to solve less than two problems, is

Options:

A. $\frac{201}{5}\left(\frac{1}{5}\right)^{49}$

B. $\frac{316}{25}\left(\frac{4}{5}\right)^{48}$

C. $\frac{54}{5}\left(\frac{4}{5}\right)^{49}$

D. $\frac{164}{25}\left(\frac{1}{5}\right)^{48}$

Answer: C

Solution:

$q =$ Probability that the candidate can solve any problem $= \frac{4}{5}$

$p = 1 - \frac{4}{5} = \frac{1}{5}$

Also, $n = 50$

$\therefore$ Required probability $= P(X < 2)$

$$= P(X = 0) + P(X = 1)$$
$$= {}^{50}C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{50} + {}^{50}C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^{49}$$
$$= \left(\frac{4}{5}\right)^{50} + 50\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^{49}$$
$$= \left(\frac{4}{5} + \frac{50}{5}\right)\left(\frac{4}{5}\right)^{49}$$
$$= \left(\frac{54}{5}\right)\left(\frac{4}{5}\right)^{49}$$

Question 4

General solution of the differential equation
$\cos x(1+\cos y)\mathrm{d}x-\sin y(1+\sin x)\mathrm{d}y=0$ **is**

Options:

A. $(1+\cos x)(1+\sin y)=\mathrm{c}$, where c is a constant of integration.

B. $1+\sin x+\cos y=\mathrm{c}$, where c is a constant of integration.

C. $(1+\sin x)(1+\cos y)=\mathrm{c}$, where c_{is} constant of integration.

D. $1+\sin x\cos y=\mathrm{c}$, where c is a constant of integration.

Answer: C

Solution:

$$\cos x(1+\cos y)\mathrm{d}x-\sin y(1+\sin x)\mathrm{d}y=0$$
$$\Rightarrow \frac{\cos x}{1+\sin x}\mathrm{d}x-\frac{\sin y}{1+\cos y}\mathrm{d}y=0$$

Integrating on both sides, we get

$$\log|1+\sin x|+\log|1+\cos y|=\log|\mathrm{c}|$$
$$\Rightarrow \log|(1+\sin x)(1+\cos y)|=\log|c|$$
$$\Rightarrow (1+\sin x)(1+\cos y)=\mathrm{c}$$

Question 5

The variance of 20 observations is 5. If each observation is multiplied by 2, then variance of resulting observations is

Options:

A. 5

B. 10

C. 4

D. 20

Answer: D

Solution:

Let's denote the original set of observations as \(X = \{x_1, x_2, ..., x_{20}\} \). The variance of these observations (\(Var(X)\)) is given as 5. Variance is a measure of how spread out the values in a dataset are, and it is calculated as the average of the squared differences from the mean:

$$Var(X) = \frac{\sum_{i=1}^{20}(x_i-\mu)^2}{20}$$

where μ represents the mean of the original observations.

Now, if each observation is multiplied by 2, we get a new set of observations $Y = \{2x_1, 2x_2, \ldots, 2x_{20}\}$. The new variance ($Var(Y)$) can be calculated as follows:

$$Var(Y) = \frac{1}{20}\sum_{i=1}^{20}(2x_i - \mu')^2$$

where μ' is the mean of the new observations. Since the mean of the new observations would also be twice the mean of the original observations (because each term in the sum is doubled), we have $\mu' = 2\mu$.

Now let's substitute and note that $(2x_i - 2\mu)^2 = 4(x_i - \mu)^2$, which gives us:

$$Var(Y) = \frac{1}{20}\sum_{i=1}^{20}4(x_i - \mu)^2$$

$$Var(Y) = 4 \times \frac{1}{20}\sum_{i=1}^{20}(x_i - \mu)^2$$

$$Var(Y) = 4 \times Var(X)$$

Since we know $Var(X) = 5$, we can now calculate $Var(Y)$:

$$Var(Y) = 4 \times 5 = 20$$

Therefore, the correct answer is

Option D : 20

Question 6

The statement $[(p \to q) \wedge \sim q] \to r$ is tautology, when r is equivalent to

Options:

A. p$\wedge \sim$ q

B. $q \vee p$

C. p $\wedge$ q

D. $\sim q$

Answer: D

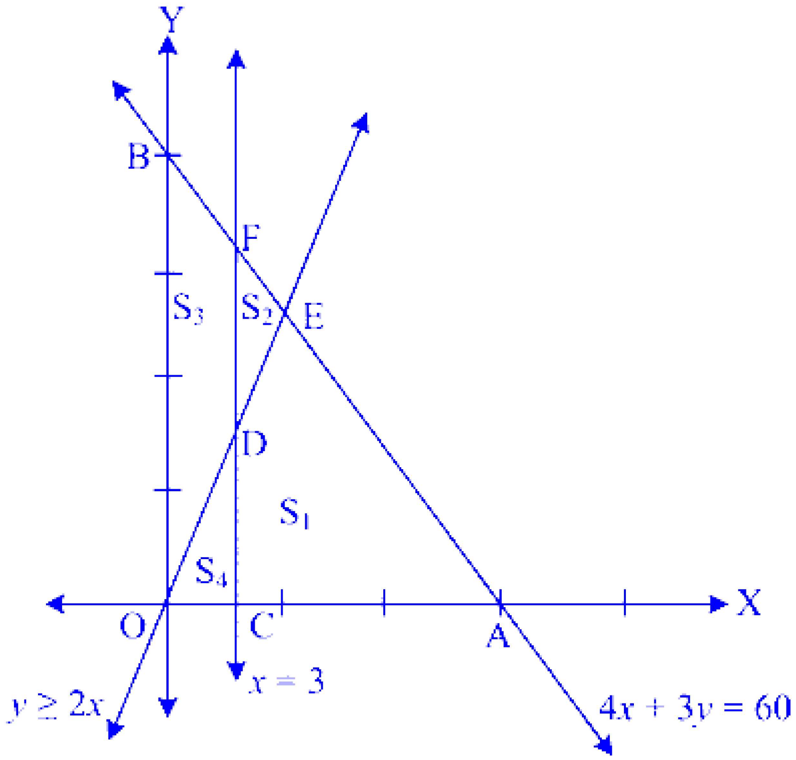
Solution:

p	q	r	$p \rightarrow q$	$\sim q$	$(p \rightarrow q) \wedge \sim q$	$[(p \rightarrow q) \wedge \sim q] \rightarrow r$
T	T	T	T	F	F	T
T	T	F	T	F	F	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	F	F	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	F

$\therefore \quad [(p \rightarrow q) \wedge \sim q] \rightarrow r$ is a tautology when all the entries in the last column are T, which is only possible when $r \equiv \sim q$

Question 7

The solution set of the inequalities $4x + 3y \leq 60, y \geq 2x, x \geq 3, x, y \geq 0$ is represented by region



Options:

A. S_2 region

B. S_1 region

C. S_3 region

D. S_4 region

Answer: A

Solution:

Take a test point $(4, 10)$ that lies within the S_2 region.

Since $4(4) + 3(10) = 46 \leq 60, 10 \geq 2(4) = 8, 4 \geq 3, 4 \geq 0, 10 \geq 0$

$\therefore$ The solution set is represented by S_2 region.

Question 8

If the line $x - 2y = \mathrm{m}(\mathrm{m} \in \mathrm{Z})$ intersects the circle $x^2 + y^2 = 2x + 4y$ at two distinct points, then the number of possible values of m are

Options:

A. 8

B. 9

C. 10

D. 11

Answer: B

Solution:

Centre of circle is ,$(1, 2)$ and

radius $= \sqrt{1+4-0} = \sqrt{5}$

Since the line intersects the circle at two points, length of perpendicular from the centre $<$ radius

$$\Rightarrow \left|\frac{1-2(2)-m}{\sqrt{1+4}}\right| < \sqrt{5}$$
$$\Rightarrow |m+3| < 5$$
$$\Rightarrow -5 < m+3 < 5$$
$$\Rightarrow -8 < m < 2$$

$\therefore$ The number of possible values of $m = 9$

Question 9

If $\bar{a}, \bar{b}, \bar{c}$ are three vectors such that $|\bar{a}+\bar{b}+\bar{c}| = 1, \bar{c} = \lambda(\bar{a} \times \bar{b})$ and $|\bar{a}| = \frac{1}{\sqrt{3}}, |\bar{b}| = \frac{1}{\sqrt{2}}, |\bar{c}| = \frac{1}{\sqrt{6}}$, then the angle between $\bar{a}$ and $\bar{b}$ is

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

Solution:

Let θ be the angle between $\bar{a}$ and $\bar{b}$.

Since $\bar{c} = \lambda(\bar{a} \times \bar{b})$

$$\Rightarrow \bar{c} \perp \bar{a}, \bar{c} \perp \bar{b}$$
$$\Rightarrow \bar{c} \cdot \bar{a} = \bar{c} \cdot \bar{b} = 0$$

Now,

$$|\overline{\mathrm{a}} + \overline{\mathrm{b}} + \overline{\mathrm{c}}| = 1$$
$$\Rightarrow |\overline{\mathrm{a}} + \overline{\mathrm{b}} + \overline{\mathrm{c}}|^2 = 1$$
$$\Rightarrow |\overline{\mathrm{a}}|^2 + |\overline{\mathrm{b}}|^2 + |\overline{\mathrm{c}}|^2 + 2(\overline{\mathrm{a}} \cdot \overline{\mathrm{b}} + \overline{\mathrm{b}} \cdot \overline{\mathrm{c}} + \overline{\mathrm{c}} \cdot \overline{\mathrm{a}}) = 1$$
$$\Rightarrow \frac{1}{3} + \frac{1}{2} + \frac{1}{6} + 2\{|\overline{\mathrm{a}}||\overline{\mathrm{b}}| \cos\theta\} = 1$$
$$\Rightarrow \cos\theta = 0$$
$$\Rightarrow \theta = \frac{\pi}{2}$$

Question 10

The equation $x^3 + x - 1 = 0$ has

Options:

A. no real root.

B. exactly two real roots.

C. exactly one real root.

D. more than two real roots.

Answer: C

Solution:

Let $\mathrm{f}(x) = x^3 + x - 1$

A root of $\mathrm{f}(x)$ exists, if $\mathrm{f}(x) = 0$ for at least one value of x.

$f(0) = -1 < 0$
$f(1) = 1 > 0$

$\therefore$ By intermediate value theorem, there has to be a point 'c' between 0 and 1 such that $f(x) = 0$.

$\therefore$ The given equation has exactly one real root.

Alternate Method:

Let $f(x) = x^3 + x - 1$

$\therefore$ $f'(x) = 3x^2 + 1$

$\Rightarrow f'(x) > 0 \quad \forall x \in R$

$\Rightarrow f(x)$ is an increasing function.

$\Rightarrow f(x)$ intersects X-axis at only one point.

$\therefore$ The given equation has exactly one real root.

Question 11

Let $\bar{a}, \bar{b}, \bar{c}$ be three vectors such that $|\bar{a}| = \sqrt{3}, |\bar{b}| = 5, \bar{b} \cdot \bar{c} = 10$ and the angle between $\bar{b}$ and $\bar{c}$ is $\frac{\pi}{3}$. If $\bar{a}$ is perpendicular to the vector $\bar{b} \times \bar{c}$, then $|\bar{a} \times (\bar{b} \times \bar{c})|$ is equal to

Options:

A. $10\sqrt{3}$

B. $5\sqrt{3}$

C. 60

D. 30

Answer: D

Solution:

$\overline{b} \cdot \overline{c} = 10$

$\Rightarrow |\overline{b}||\overrightarrow{c}| \cos \frac{\pi}{3} = 10$

$\Rightarrow (s)|\overline{c}| \left(\frac{1}{2}\right) = 10$

$\Rightarrow |\overline{c}| = 4$

$|\overline{a} \times (\overline{b} \times \overline{c})| = |\overline{a}||\overline{b} \times \overline{c}| \sin \frac{\pi}{2}$

$= |\overline{a}||\overline{b} \times \overline{c}|$

$= |\overline{a}||\overline{b}||\overline{c}| \sin \frac{\pi}{3}$

$= (\sqrt{3})(5)(4) \left(\frac{\sqrt{3}}{2}\right)$

$= 30$

Question 12

Let $A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$. If $B = I - {}^3C_1(\text{adj } A) + {}^3C_2(\text{adj } A)^2 - {}^3C_3(\text{adj } A)^3$, then the sum of all elements of the matrix B is

Options:

A. -1

B. -3

C. -4

D. -5

Answer: D

Solution:

$B = I - {}^3C_1(\text{adj } A) + {}^3C_2(\text{adj } A)^2 - {}^3C_3(\text{adj } A)^3$
$= I - 3 \text{ adj } A + 3(\text{adj } A)^2 - 1(\text{adj } A)^3$
$= (I - \text{adj } A)^3$

$\text{adj } A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

$\therefore \quad B = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \right)^3$

$= \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}^3$

$= \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} -1 & -3 \\ 0 & -1 \end{bmatrix}$

Sum of all elements of the matrix B

$= -1 - 3 - 1 = -5$

Question 13

If $\triangle \mathrm{ABC}$ is right angled at A, where $A \equiv (4, 2, x), \mathrm{B} \equiv (3, 1, 8)$ and $C \equiv (2, -1, 2)$, then the value of x is

Options:

A. 4

B. 2

C. 3

D. 1

Answer: C

Solution:

Since $\triangle ABC$ is right angled at A,

$\overline{\mathrm{AB}} \cdot \overline{\mathrm{AC}} = 0$

$\Rightarrow [-\hat{i}-\hat{j}+(8-x)\hat{k}]\cdot(-2\hat{i}-3\hat{j}+(2-x)\hat{k})=0$
$\Rightarrow 2+3+(8-x)(2-x)=0$
$\Rightarrow x^2-10x+21=0$
$\Rightarrow (x-3)(x-7)=0$
$\Rightarrow x=3$ or $x=7$

Question 14

The angle between the lines, whose direction cosines $l,\ \mathrm{m}, \mathrm{n}$ satisfy the equations $l+\mathrm{m}+\mathrm{n}=0$ and $2l^2+2\,\mathrm{m}^2-\mathrm{n}^2=0$, is

Options:

A. 60°

B. 180°

C. 90°

D. 30°

Answer: B

Solution:

Substituting $\mathrm{n}=-l-\mathrm{m}$ in $2l^2+2m^2-\mathrm{n}^2=0$, we get

$2l^2+2\,\mathrm{m}^2-(-l-\mathrm{m})^2=0$

$\Rightarrow l^2+\mathrm{m}^2-2l\,\mathrm{m}=0$

$\Rightarrow (l-\mathrm{m})^2=0$

$\Rightarrow l=\mathrm{m}$

If $l=\mathrm{m}$, then $\mathrm{n}=-2\,\mathrm{m}$

$\Rightarrow \frac{l}{1}=\frac{\mathrm{m}}{1}=\frac{\mathrm{n}}{-2}$

The direction ratios of both the lines are same.

$\therefore \quad \cos\theta=\pm 1$
$\Rightarrow \theta=0^\circ$ or 180°

Question 15

Let $f: R \to R$ be a function such that $\mathrm{f}(x)=x^3+x^2\mathrm{f}'(1)+x\mathrm{f}''(2)+6, x \in \mathrm{R}$, then $\mathrm{f}(2)$ equals

Options:

A. 30

B. -4

C. -2

D. 8

Answer: C

Solution:

$$\begin{aligned} & \mathrm{f}(x)=x^3+x^2\mathrm{f}'(1)+x\mathrm{f}''(2)+6 \\ \therefore\ & \mathrm{f}'(x)=3x^2+2x\mathrm{f}'(1)+\mathrm{f}''(2) \quad \text{.... (i)} \\ \therefore\ & \mathrm{f}''(x)=6x+2\mathrm{f}'(1) \quad \text{..... (ii)} \end{aligned}$$

Substituting $x=1$ in (i), we get

$$f'(1)=3(1)^2+2(1)f'(1)+f''(2)$$
$$\Rightarrow f'(1)+f''(2)=-3 \quad \text{..... (iii)}$$

Substituting $x=2$ in (ii), we get

$$f''(2)=6(2)+2f'(1)$$
$$\Rightarrow f''(2)=12+2f'(1) \quad \text{..... (iv)}$$

From (iii) and (iv), we get

$$\begin{aligned} & \mathrm{f}'(1)+12+2\mathrm{f}'(1)=-3 \\ & \Rightarrow 3\mathrm{f}'(1)=-15 \\ & \Rightarrow \mathrm{f}'(1)=-5 \\ & \text{From (iii), } -5+\mathrm{f}''(2)=-3 \\ & \Rightarrow \mathrm{f}''(2)=2 \\ & \therefore\quad \mathrm{f}(2)=2^3+2^2(-5)+2(2)+6 \\ & =8-20+4+6 \\ & =-2 \end{aligned}$$

Question 16

If $\sin(\theta-\alpha)$, $\sin\theta$ and $\sin(\theta+\alpha)$ are in H.P., then the value of $\cos 2\theta$ is

Options:

A. $1+4\cos^2\frac{\alpha}{2}$

B. $1-4\cos^2\frac{\alpha}{2}$

C. $-1-4\cos^2\frac{\alpha}{2}$

D. $-1+4\cos^2\frac{\alpha}{2}$

Answer: B

Solution:

$\sin(\theta-\alpha), \sin\theta$ and $\sin(\theta+\alpha)$ are in H.P.

$\Rightarrow \frac{1}{\sin(\theta-\alpha)}, \frac{1}{\sin\theta}, \frac{1}{\sin(\theta+\alpha)}$ will be in A.P.

$$\therefore \quad \frac{2}{\sin\theta}=\frac{1}{\sin(\theta-\alpha)}+\frac{1}{\sin(\theta+\alpha)}$$
$$\Rightarrow \frac{2}{\sin\theta}=\frac{\sin(\theta+\alpha)+\sin(\theta-\alpha)}{\sin(\theta-\alpha)\sin(\theta+\alpha)}$$
$$\Rightarrow \frac{2}{\sin\theta}=\frac{2\sin\theta\cos\alpha}{\sin^2\theta-\sin^2\alpha}$$
$$\Rightarrow \sin^2\theta-\sin^2\alpha=\sin^2\theta\cos\alpha$$
$$\Rightarrow \sin^2\theta(1-\cos\alpha)=\sin^2\alpha$$
$$\Rightarrow \sin^2\theta\left(2\sin^2\frac{\alpha}{2}\right)=4\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}$$
$$\Rightarrow 1-\cos^2\theta=2\cos^2\frac{\alpha}{2}$$
$$\Rightarrow \cos^2\theta=1-2\cos^2\frac{\alpha}{2}$$
$$\Rightarrow 2\cos^2\theta-1=1-4\cos^2\frac{\alpha}{2}$$
$$\Rightarrow \cos^2\theta=1-4\cos^2\frac{\alpha}{2}$$

Question 17

If $y=\left(\sin^{-1}x\right)^2+\left(\cos^{-1}x\right)^2$, then $\left(1-x^2\right)y_2-xy_1=$

Options:

A. 1

B. 4

C. -4

D. -1

Answer: B

Solution:

$$
\begin{aligned}
& y=\left(\sin^{-1}x\right)^2+\left(\cos^{-1}x\right)^2 \\
\therefore\quad & \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\sin^{-1}x}{\sqrt{1-x^2}}-\frac{2\cos^{-1}x}{\sqrt{1-x^2}} \\
& \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\left(\sin^{-1}x-\cos^{-1}x\right)}{\sqrt{1-x^2}} \\
& \Rightarrow \sqrt{1-x^2}\frac{\mathrm{d}y}{\mathrm{d}x}=2\left(\sin^{-1}x-\cos^{-1}x\right)
\end{aligned}
$$

Differentiating both sides w.r.t. x, we get

$$
\begin{aligned}
& \sqrt{1-x^2}\cdot\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}y}{\mathrm{d}x}\cdot\frac{1}{2\sqrt{1-x^2}}\cdot(-2x) \\
& =2\left(\frac{1}{\sqrt{1-x^2}}-\frac{(-1)}{\sqrt{1-x^2}}\right)=\frac{4}{\sqrt{1-x^2}} \\
& \therefore\quad \left(1-x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-x\frac{\mathrm{d}y}{\mathrm{d}x}=4
\end{aligned}
$$

Question 18

If $a>0$ and $z=\frac{(1+i)^2}{a-i}, i=\sqrt{-1}$, has magnitude $\frac{2}{\sqrt{5}}$, then $\bar{z}$ is

Options:

A. $-\frac{2}{5}-\frac{4}{5}\mathrm{i}$

B. $-\frac{2}{5}+\frac{4}{5}\mathrm{i}$

C. $\frac{2}{5}-\frac{4}{5}\mathrm{i}$

D. $\frac{2}{5}+\frac{4}{5}\mathrm{i}$

Answer: A

Solution:

$$z=\frac{(1+i)^2}{a-i}$$
$$=\frac{2i}{a-i}=\frac{2i(a+i)}{a^2+1}$$
$$=\frac{-2+2ai}{a^2+1}$$

$$\therefore \quad |z|=\sqrt{\frac{4+4a^2}{\left(a^2+1\right)^2}}$$
$$\Rightarrow \frac{2}{\sqrt{5}}=\frac{2}{\sqrt{a^2+1}}$$
$$\Rightarrow a=2 \quad \ldots[\because a>0]$$
$$\therefore \quad z=\frac{-2+4i}{5}=\frac{-2}{5}+\frac{4}{5}i$$
$$\Rightarrow \bar{z}=-\frac{2}{5}-\frac{4}{5}i$$

Question 19

In $\triangle \mathrm{ABC}$, with usual notations, $2\mathrm{ac}\sin\left(\frac{1}{2}(\mathrm{A}-\mathrm{B}+\mathrm{C})\right)$ is equal to

Options:

A. $\mathrm{a}^2+\mathrm{b}^2-\mathrm{c}^2$

B. $\mathrm{c}^2+\mathrm{a}^2-\mathrm{b}^2$

C. $\mathrm{b}^2-\mathrm{c}^2-\mathrm{a}^2$

D. $\mathrm{c}^2-\mathrm{a}^2-\mathrm{b}^2$

Answer: B

Solution:

$$\begin{aligned}2ac\sin\frac{A-B+C}{2} &= 2ac\sin\frac{\pi-2B}{2}\\ &= 2ac\cos B\\ &= 2ac\frac{c^2+a^2-b^2}{2ca} \quad \text{...[By cosine rule]}\\ &= c^2+a^2-b^2\end{aligned}$$

Question 20

$$\int \frac{\sin 2x\left(1-\frac{3}{2}\cos x\right)}{e^{\sin^2 x+\cos^3 x}}dx =$$

Options:

A. $e^{\sin^2 x+\cos^3 x}+c$, where c is a constant of integration.

B. $-e^{-(\sin^2 x+\cos^3 x)}+c$, where c is a constant of integration.

C. $e^{-(\sin^2 x+\cos^3 x)^2}+c$, where c is a constant of integration.

D. $e^{\sin^2 x+\cos x}+c$, where c is a constant of integration.

Answer: B

Solution:

Put $\sin^2 x+\cos^3 x=\mathrm{t}$

$\Rightarrow\left(2 \sin x \cos x-3 \cos^2 x \sin x\right) \mathrm{d} x=\mathrm{dt}$

$\Rightarrow\left(\sin 2 x-\frac{3}{2} \sin 2 x \cos x\right) \mathrm{d} x=\mathrm{dt}$

$\Rightarrow \sin 2 x\left(1-\frac{3}{2} \cos x\right) \mathrm{d} x=\mathrm{dt}$

$\therefore \quad \int \frac{\sin 2 x\left(1-\frac{3}{2} \cos x\right)}{\mathrm{e}^{\sin^2 x+\cos^3 x}} \mathrm{~d} x$

$=\int \frac{1}{\mathrm{e}^{\mathrm{t}}} \mathrm{dt}$

$=\int \mathrm{e}^{-t} \mathrm{dt}$

$=-\mathrm{e}^{-t}+\mathrm{c}$

$=-\mathrm{e}^{-\left(-\sin^2 x+\cos^3 x\right)}+\mathrm{c}$

Question 21

If $\mathrm{f}'(x)=\tan^{-1}(\sec x+\tan x),-\frac{\pi}{2}<x<\frac{\pi}{2}$ and $f(0)=0$, then $\mathrm{f}(1)$ is

Options:

A. $\frac{\pi+1}{4}$

B. $\frac{\pi+2}{4}$

C. $\pi+\frac{1}{4}$

D. $\frac{\pi-1}{4}$

Answer: A

Solution:

$$f'(x) = \tan^{-1}(\sec x + \tan x)$$
$$= \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$$
$$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right]$$
$$= \tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)$$
$$= \tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$$
$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$$

$$\therefore \quad f'(x) = \frac{\pi}{4}+\frac{x}{2}$$
$$\Rightarrow f(x) = \int\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$$
$$= \frac{\pi x}{4}+\frac{1}{2}\cdot\frac{x^2}{2}+c$$

$$\therefore \quad f(0) = c$$
$\Rightarrow c = 0$ ....[$\because$ f(0)=0 (given)]
$$\therefore \quad f(x) = \frac{\pi x}{4}+\frac{x^2}{4}$$
$$\Rightarrow f(1) = \frac{\pi+1}{4}$$

Question 22

If $\int \frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta = A\log_e|f(\theta)|+c$ (where c is a constant of integration), then $\frac{f(\theta)}{A}$ can be

Options:

A. $\frac{2\sin\theta+1}{\sin\theta+3}$

B. $\frac{2\sin\theta+1}{5(\sin\theta+3)}$

C. $\frac{5(\sin\theta+3)}{2\sin\theta+1}$

D. $\frac{5(2\sin\theta+1)}{\sin\theta+3}$

Answer: D

Solution:

$$\text{Let } I=\int \frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta$$
$$=\int \frac{\cos\theta}{5+7\sin\theta-2\left(1-\sin^2\theta\right)}d\theta$$
$$=\int \frac{\cos\theta}{2\sin^2\theta+7\sin\theta+3}d\theta$$
$$=\int \frac{\cos\theta}{(\sin\theta+3)(2\sin\theta+1)}d\theta$$

Put $\sin\theta=t$

$\Rightarrow \cos\theta d\theta=dt$

$$\therefore \quad I=\int \frac{dt}{(t+3)(2t+1)}$$
$$=\int\left[\frac{2}{5(2t+1)}-\frac{1}{5(t+3)}\right]dt$$
$$=\frac{2}{5}\cdot\frac{\log|2t+1}{2}-\frac{1}{5}\log|t+3|+c$$
$$=\frac{1}{5}\log|2\sin\theta+1|-\frac{1}{5}\log|\sin\theta+3|+c$$
$$=\frac{1}{5}\log\left|\frac{2\sin\theta+1}{\sin\theta+3}\right|+c$$

$$\therefore \quad \mathrm{A}=\frac{1}{5},\mathrm{f}(\theta)=\frac{2\sin\theta+1}{\sin\theta+3}$$
$$\therefore \quad \frac{\mathrm{f}(\theta)}{\mathrm{A}}=\frac{5(2\sin\theta+1)}{\sin\theta+3}$$

Question 23

If $\overline{\mathrm{a}},\overline{\mathrm{b}},\overline{\mathrm{c}}$ are unit vectors and θ is angle between $\overline{\mathrm{a}}$ and $\bar{c}$ and $\bar{a}+2\bar{b}+2\bar{c}=\overline{0}$, then $|\bar{a}\times\bar{c}|=$

Options:

A. $\frac{\sqrt{15}}{2}$

B. $\frac{\sqrt{15}}{4}$

C. $\sqrt{15}$

D. $\frac{\sqrt{15}}{3}$

Answer: B

Solution:

$\bar{a}+2\bar{b}+2\bar{c}=\overline{0}$
$\Rightarrow a+2\bar{c}=-2\bar{b}$

Squaring on both sides, we get

$|\overline{\mathrm{a}}|^2+4\overline{\mathrm{a}}\cdot\overline{\mathrm{c}}+4|\overline{\mathrm{c}}|^2=4|\overline{\mathrm{b}}|^2$
$\Rightarrow 1+4|\overrightarrow{\mathrm{a}}||\overrightarrow{\mathrm{c}}|\cos\theta+4=4$
$\Rightarrow \cos\theta=-\frac{1}{4}$
$\Rightarrow \sin\theta=\frac{\sqrt{15}}{4}$
$|\overline{\mathrm{a}}\times\overline{\mathrm{c}}|=|\overline{\mathrm{a}}||\overline{\mathrm{c}}|\sin\theta$
$=(1)(1)\left(\frac{\sqrt{15}}{4}\right)=\frac{\sqrt{15}}{4}$

Question 24

The integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sec^{\frac{2}{3}}x\operatorname{cosec}^{\frac{4}{3}}x\,dx$ is equal to

Options:

A. $3^{\frac{5}{6}}-3^{\frac{2}{3}}$

B. $3^{\frac{7}{6}}-3^{\frac{5}{6}}$

C. $3^{\frac{5}{3}}-3^{\frac{1}{3}}$

D. $3^{\frac{4}{3}}-3^{\frac{1}{3}}$

Answer: B

Solution:

Let $I = \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x \, dx$

$$= \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\cos^{\frac{2}{3}} x \cdot \sin^{\frac{4}{3}} x}$$

$$= \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\frac{\sin^{\frac{4}{3}} x}{\cos^{\frac{4}{3}} x} \cdot \cos^2 x}$$

$$= \int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan^{\frac{4}{3}} x} dx$$

Put $\tan x = \mathrm{t}$

$\Rightarrow \sec^2 x \, dx = \mathrm{dt}$

$\therefore \quad I = \int\limits_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{dt}{t^{\frac{4}{3}}}$

$$= \left[-3\mathrm{t}^{-\frac{1}{3}}\right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}$$

$$= -3\left[(\sqrt{3})^{\frac{-1}{3}} - \left(\frac{1}{\sqrt{3}}\right)^{\frac{-1}{3}}\right]$$

$$= -3\left(3^{\frac{1}{6}} - 3^{\frac{1}{6}}\right)$$

$$= 3^{\frac{7}{6}} - 3^{\frac{5}{6}}$$

Question 25

The principal solutions of the equation $\sec x + \tan x = 2\cos x$ are

Options:

A. $\frac{\pi}{6}, \frac{5\pi}{6}$

B. $\frac{\pi}{6}, \frac{\pi}{20}$

C. $\frac{\pi}{6}, \frac{2\pi}{3}$

D. $\frac{\pi}{6}, \frac{\pi}{12}$

Answer: A

Solution:

The given equation is defined for $x \neq \frac{\pi}{2}, \frac{3\pi}{2}$.

Now, $\tan x + \sec x = 2\cos x$

$$\Rightarrow \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$$
$$\Rightarrow (\sin x + 1) = 2\cos^2 x$$
$$\Rightarrow (\sin x + 1) = 2\left(1 - \sin^2 x\right)$$
$$\Rightarrow (\sin x + 1) = 2(1 - \sin x)(1 + \sin x)$$
$$\Rightarrow (1 + \sin x)[2(1 - \sin x) - 1] = 0$$
$$\Rightarrow 2(1 - \sin x) - 1 = 0$$

$$\ldots \left[\because \sin x \neq -1 \text{ otherwise } \cos x = 0 \text{ and } \tan x, \sec x \text{ will be undefined}\right]$$

$$\Rightarrow \sin x = \frac{1}{2}$$
$$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6} \text{ in } (0, 2\pi)$$

Question 26

If $\bar{a}, \bar{b}, \bar{c}$ are three vectors with magnitudes $\sqrt{3}$, 1, 2 respectively, such that $\bar{a} \times (\bar{a} \times \bar{c}) + 3\bar{b} = \bar{0}$, if θ is the angle between $\bar{a}$ and $\bar{c}$, then $\sec^2\theta$ is

Options:

A. 1

B. $\frac{3}{2}$

C. $\frac{4}{3}$

D. $\frac{2}{\sqrt{3}}$

Answer: C

Solution:

Given $|\bar{a}|=\sqrt{3}, |\bar{b}|=1, |\bar{c}|=2$

$\bar{a} \times (\bar{a} \times \bar{c})+3\bar{b}=\overline{0}$

$\Rightarrow (\bar{a} \cdot \bar{c})\bar{a}-(\bar{a} \cdot \bar{a})\bar{c}+3\bar{b}=\overline{0}$

$\Rightarrow (\bar{a} \cdot \bar{c})\bar{a}-3\bar{c}=-3\bar{b} \quad \ldots\left[\bar{a} \cdot \bar{a}=|\bar{a}|^2=3\right]$

$\Rightarrow |(\bar{a} \cdot \bar{c})\bar{a}-3\bar{c}|=|-3\bar{b}|$

$\Rightarrow |(\bar{a} \cdot \bar{c})\bar{a}-3\bar{c}|^2=|-3\bar{b}|^2$

$\Rightarrow |(\bar{a} \cdot \bar{c})\bar{a}|^2+9|\bar{c}|^2-6\{(\bar{a} \cdot \bar{c})(\bar{a} \cdot \bar{c})\}=9|\bar{b}|^2$

$\Rightarrow (\bar{a} \cdot \bar{c})^2|\bar{a}|^2+9|\bar{c}|^2-6(\bar{a} \cdot \bar{c})(\bar{a} \cdot \bar{c})=9|\bar{b}|^2$

$\Rightarrow (\bar{a} \cdot \bar{c})^2\left(|\vec{a}|^2-6\right)+9|\bar{c}|^2=9|\bar{b}|^2$

$\Rightarrow (\bar{a} \cdot \bar{c})^2(3-6)+9(2)^2=9(1)^2$

$\Rightarrow -3(\bar{a} \cdot \bar{c})^2=-27$

$\Rightarrow (\bar{a} \cdot \bar{c})^2=9$

$\Rightarrow \bar{a} \cdot \bar{c}=\pm 3$

$\Rightarrow |\vec{a}||\vec{c}| \cos \theta=\pm 3$

$\Rightarrow (\sqrt{3})(2) \cos \theta=\pm 3$

$\Rightarrow \cos \theta=\pm \frac{\sqrt{3}}{2}$

$\Rightarrow \sec \theta=\pm \frac{2}{\sqrt{3}}$

$\Rightarrow \sec^2 \theta=\frac{4}{3}$

Question 27

Let the curve be represented by $x=2(\cos t+t \sin t), y=2(\sin t-t \cos t)$. Then normal at any point 't' of the curve is at a distance of ______ units from the origin.

Options:

A. 1

B. 0

C. 2

D. 4

Answer: C

Solution:

$$\begin{aligned} & x=2(\cos t+t \sin t) \\ \therefore \quad & \frac{dx}{dt}=2(-\sin t+\sin t+t \cos t)=2t \cos t \\ & y=2(\sin t-t \cos t) \\ \therefore \quad & \frac{dy}{dt}=2(\cos t-\cos t+t \sin t)=2t \sin t \\ \therefore \quad & \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2t \sin t}{2t \cos t}=\tan t \end{aligned}$$

$$\text{Slope of normal }=-\frac{1}{\frac{dy}{dx}}=-\frac{1}{\tan t}=-\frac{\cos t}{\sin t}$$

$\therefore$ Equation of the normal is

$$\begin{aligned} & y-2(\sin t-t \cos t)=-\frac{\cos t}{\sin t}[x-2(\cos t+t \sin t)] \\ & \Rightarrow y \sin t-2 \sin^2 t+2t \sin t \cos t \\ & \quad=-x \cos t+2 \cos^2 t+2t \sin t \cos t \\ & \Rightarrow x \cos t+y \sin t=2\left(\sin^2 t+\cos^2 t\right) \\ & \Rightarrow x \cos t+y \sin t=2 \end{aligned}$$

$$\therefore \quad \text{Distance from origin }=\left|\frac{-2}{\sqrt{\cos^2 t+\sin^2 t}}\right|=2 \text{ units}$$

Question 28

Equation of the plane passing through $(1,-1,2)$ and perpendicular to the planes $x+2y-2z=4$ and $3x+2y+z=6$ is

Options:

A. $6x-7y-4z-5=0$

B. $6x+7y-4z+5=0$

C. $6x-7y+4z+5=0$

D. $6x+7y+4z-5=0$

Answer: A

Solution:

The equation of plane passing through $(1,-1,2)$ is

a$(x-1)+$b$(y+1)+$c(z $-2)=0$ ..... (i)

Since plane (i) is perpendicular to the planes $x+2y-2z=4$ and $3x+2y+z=6$

$\therefore \quad a+2b-2c=0$
and $3a+2b+c=0$
$\Rightarrow \frac{a}{6}=\frac{b}{-7}=\frac{c}{-4}$

$\therefore$ The equation of the required plane is

$6(x-1)-7(y+1)-4(z-2)=0$
$\Rightarrow 6x-7y-4z-5=0$

Question 29

If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$, then the value of $x^{2025}+x^{2026}+x^{2027}$ is

Options:

A. -1

B. 0

C. 1

D. 3

Answer: A

Solution:

$\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$

Since $0\leq\cos^{-1}x\leq\pi$,

$0\leq\cos^{-1}y\leq\pi$ and $0\leq\cos^{-1}\mathrm{z}\leq\pi$

Here, $\cos^{-1}x=\cos^{-1}y=\cos^{-1}z=\pi$

$\Rightarrow x=y=\mathrm{z}=\cos\pi=-1$

$\therefore\quad x^{2005}+x^{2026}+x^{2027}$

$=(-1)^{2025}+(-1)^{2026}+(-1)^{2027}$

$=-1+1-1$

$=-1$

Question 30

p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b respectively, then $\frac{1}{\mathrm{a}^2}+\frac{1}{\mathrm{b}^2}$ equals

Options:

A. p^2

B. $\frac{2}{\mathrm{p}^2}$

C. $\frac{1}{\mathrm{p}^2}$

D. $\frac{1}{2\mathrm{p}^2}$

Answer: C

Solution:

Let the equation of the line be

$\frac{x}{a}+\frac{y}{b}=1$

According to the given condition,

$$p=\left|\frac{ab}{\sqrt{a^2+b^2}}\right|$$
$$\Rightarrow \frac{a^2+b^2}{a^2b^2}=\frac{1}{p^2}$$
$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}$$

Question 31

If $f(x)=\begin{cases}3\left(1-2x^2\right) & ;0<x<1\\ 0 & ;\ \text{otherwise}\end{cases}$ **is a probability density function of X, then** $\mathrm{P}\left(\frac{1}{4}<x<\frac{1}{3}\right)$ **is**

Options:

A. $\frac{75}{243}$

B. $\frac{23}{96}$

C. $\frac{179}{864}$

D. $\frac{52}{243}$

Answer: C

Solution:

$$P\left(\frac{1}{4}<x<\frac{1}{3}\right)=\int_{\frac{1}{4}}^{\frac{1}{3}}\mathrm{f}(x)\mathrm{d}x=\int_{\frac{1}{4}}^{\frac{1}{3}}3\left(1-2x^2\right)\mathrm{d}x$$

$$=\left[3x-2x^3\right]_{\frac{1}{4}}^{\frac{1}{3}}$$
$$=\left(1-\frac{2}{27}\right)-\left(\frac{3}{4}-\frac{1}{32}\right)$$
$$=\frac{1}{4}+\frac{1}{32}-\frac{2}{27}$$
$$=\frac{179}{864}$$

Question 32

$\int \frac{\sin x+\sin^3 x}{\cos 2x} dx = A\cos x + B\log \mathrm{f}(x) + c$ **(where** c **is a constant of integration). Then values of** A, B **and** $\mathrm{f}(x)$ **are**

Options:

A. $\mathrm{A} = \frac{1}{2}$, $\mathrm{B} = \frac{-3}{4\sqrt{2}}, \mathrm{f}(x) = \frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}$

B. $A = -\frac{1}{2}, B = \frac{-3}{4\sqrt{2}}, \mathrm{f}(x) = \frac{\sqrt{2}\cos x+1}{\sqrt{2}\cos x-1}$

C. $\mathrm{A} = \frac{1}{2}$, $\mathrm{B} = \frac{-3}{4\sqrt{2}}, \mathrm{f}(x) = \frac{\sqrt{2}\cos x+1}{\sqrt{2}\cos x-1}$

D. $\mathrm{A} = \frac{3}{2}$, $\mathrm{B} = \frac{1}{2}, \mathrm{f}(x) = \frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}$

Answer: A

Solution:

$$\begin{aligned}\text{Let } I &= \int \frac{\sin x+\sin^3 x}{\cos 2x} dx \\ &= \int \frac{\sin x\left(1+\sin^2 x\right)}{\cos 2x} dx \\ &= \int \frac{\sin x\left(1+1-\cos^2 x\right)}{2\cos^2 x-1} dx \\ &= \int \frac{\sin x\left(2-\cos^2 x\right)}{2\cos^2 x-1} dx\end{aligned}$$

Put $\cos x = \mathrm{t} \Rightarrow \sin x \, \mathrm{d}x = -\mathrm{dt}$

$$\therefore \quad I = -\int \frac{2-t^2}{2t^2-1} dt$$
$$= \int \frac{t^2-2}{2t^2-1} dt$$
$$= \frac{1}{2} \int \frac{2t^2-4}{2t^2-1} dt$$
$$= \frac{1}{2} \int \left(1 - \frac{3}{2t^2-1}\right) dt$$
$$= \frac{1}{2} \int dt - \frac{3}{2} \int \frac{dt}{(\sqrt{2}t)^2 - 1^2}$$
$$= \frac{1}{2}t - \frac{3}{2} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \log \left| \frac{\sqrt{2}t-1}{\sqrt{2}t+1} \right| + c$$
$$= \frac{1}{2}\cos x - \frac{3}{4\sqrt{2}} \log \left| \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1} \right| + c$$
$$\therefore \quad A = \frac{1}{2}, B = \frac{-3}{4\sqrt{2}}, f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$$

Question 33

If $y = [(x+1)(2x+1)(3x+1)\ldots\ldots(\mathrm{n}x+1)]^n$, then $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 0$ is

Options:

A. $\frac{\mathrm{n}(\mathrm{n}+1)}{2}$

B. $\frac{\mathrm{n}^2(\mathrm{n}+1)}{2}$

C. $\frac{\mathrm{n}(\mathrm{n}+1)}{4}$

D. $\frac{\mathrm{n}^2(\mathrm{n}-1)}{2}$

Answer: B

Solution:

$y = [(x+1)(2x+1)(3x+1)\ldots(\mathrm{n}x+1)]^{\mathrm{n}}$
$\Rightarrow \log y = \mathrm{n}\log[(x+1)(2x+1)(3x+1)\ldots(\mathrm{n}x+1)]$

$\Rightarrow \log y = \mathrm{n}[\log(x+1) + \log(2x+1) + \log(3x+1) + \ldots + \log(nx+1)]$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{n}\left(\frac{1}{x+1} + \frac{2}{2x+1} + \frac{3}{3x+1} + \ldots + \frac{\mathrm{n}}{\mathrm{n}x+1}\right)$$

$$\Rightarrow \frac{1}{1} \cdot \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=0} = \mathrm{n}(1+2+3+\ldots+\mathrm{n}) \quad \ldots [\text{At } x=0, y=1]$$

$$\Rightarrow \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=0} = \mathrm{n}\left[\frac{\mathrm{n}(\mathrm{n}+1)}{2}\right] = \frac{\mathrm{n}^2(\mathrm{n}+1)}{2}$$

Question 34

Let $\mathrm{B} \equiv (0,3)$ and $\mathrm{C} \equiv (4,0)$. The point A is moving on the line $y = 2x$ at the rate of 2 units/second. The area of $\triangle \mathrm{ABC}$ is increasing at the rate of

Options:

A. $\frac{11}{\sqrt{5}}$ (units)^2/ sec

B. $\frac{11}{5}$ (units)^2/ sec

C. $\frac{13}{\sqrt{5}}$ (units)^2/ sec

D. $\frac{13}{5}$ (units)^2/ sec

Answer: A

Solution:

Let $A = (h, 2h)$

$OA = \sqrt{h^2 + 4h^2} = \sqrt{5}h$

$$\therefore \quad \frac{\mathrm{d(OA)}}{\mathrm{dt}} = \sqrt{5}\frac{\mathrm{dh}}{\mathrm{dt}}$$

$$\Rightarrow 2 = \sqrt{5}\frac{\mathrm{dh}}{\mathrm{dt}}$$

$$\Rightarrow \frac{\mathrm{dh}}{\mathrm{dt}} = \frac{2}{\sqrt{5}} \quad \text{.... (i)}$$

$$\alpha = \mathrm{A}(\triangle \mathrm{ABC}) = \frac{1}{2}\begin{vmatrix} \mathrm{h} & 2\,\mathrm{h} & 1 \\ 0 & 3 & 1 \\ 4 & 0 & 1 \end{vmatrix}$$

$$=\frac{1}{2}(3\text{ h}+8\text{ h}-12)$$
$$=\frac{11\text{ h}-12}{2}$$
$$\therefore \quad \frac{d\alpha}{dt}=\frac{11}{2}\cdot\frac{dh}{dt}$$
$$=\frac{11}{2}\cdot\frac{2}{\sqrt{5}} \quad \text{.... [From (i)]}$$
$$=\frac{11}{\sqrt{5}}(\text{units})^2/\text{sec}$$

Question 35

$$\lim_{x\to\infty} x^3\left\{\sqrt{x^2+\sqrt{1+x^4}}-x\sqrt{2}\right\}=$$

Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{4\sqrt{2}}$

C. $\frac{-1}{4\sqrt{2}}$

D. $\frac{-1}{\sqrt{2}}$

Answer: B

Solution:

$$\lim_{x\to\infty} x^3\left(\sqrt{x^2+\sqrt{1+x^4}}-x\sqrt{2}\right)$$
$$=\lim_{x\to\infty}\frac{x^3\left(x^2+\sqrt{1+x^4}-2x^2\right)}{\sqrt{x^2+\sqrt{1+x^4}}+x\sqrt{2}}$$
$$=\lim_{x\to\infty}\frac{x^3\left(\sqrt{1+x^4}-x^2\right)}{\sqrt{x^2+\sqrt{1+x^4}}+x\sqrt{2}}$$
$$=\lim_{x\to\infty}\frac{x^3(1+x^4-x^4)}{\left(\sqrt{x^2+\sqrt{1+x^4}}+x\sqrt{2}\right)\left(\sqrt{1+x^4}+x^2\right)}$$

$$= \lim_{x\to\infty} \frac{x^3}{x^3\left(\sqrt{1+\sqrt{\frac{1}{x^4}+1}+\sqrt{2}}\right)\left(\sqrt{\frac{1}{x^4}+1}+1\right)}$$

$$= \lim_{x\to\infty} \frac{1}{\left(\sqrt{1+\sqrt{\frac{1}{x^4}+1}+\sqrt{2}}\right)\left(\sqrt{\frac{1}{x^4}+1}+1\right)}$$

$$= \frac{1}{(\sqrt{1+1}+\sqrt{2})(1+1)}$$

$$= \frac{1}{4\sqrt{2}}$$

Question 36

The money invested in a company is compounded continuously. If ₹ 200 invested today becomes ₹ 400 in 6 years, then at the end of 33 years it will become ₹

Options:

A. $1600\sqrt{2}$

B. $3200\sqrt{2}$

C. $12800\sqrt{2}$

D. $6400\sqrt{2}$

Answer: D

Solution:

Here, Amount $(A) = 400$

Principal $(P) = 200, N = 6$ years

$$A = P\left(1 + \frac{R}{100}\right)^N$$

$$\Rightarrow 400 = 200\left(1 + \frac{R}{100}\right)^6$$

$$\Rightarrow \left(1 + \frac{R}{100}\right)^6 = 2$$

$$\Rightarrow 1 + \frac{R}{100} = 2^{\frac{1}{6}}$$

$$A = P\left(1 + \frac{R}{100}\right)^N$$

$$= 200\left(1 + \frac{R}{100}\right)^{33}$$

$$= 200\left(2^{\frac{1}{6}}\right)^{33}$$

$$= 200\left(2^5 \cdot 2^{\frac{1}{2}}\right)$$

$$= 200(32\sqrt{2})$$

$$= 6400\sqrt{2}$$

Question 37

The range of the function $\mathrm{f}(x) = \frac{x^2}{x^2+1}$ **is**

Options:

A. $(0, 1)$

B. $[0, 1)$

C. $(0, 1]$

D. $[0, 1]$

Answer: B

Solution:

$$\text{Let } y = \frac{x^2}{x^2+1}$$
$$\Rightarrow yx^2 + y = x^2$$
$$\Rightarrow x^2(y-1) + y = 0$$
$$\Rightarrow x^2 = \frac{y}{1-y}$$

For x to be real,

$y(1-y) \geq 0$ and $1-y \neq 0$
$\Rightarrow y(y-1) \leq 0$ and $y \neq 1$
$\Rightarrow 0 \leq y < 1$

Question 38

The differential equation of $y = \mathrm{e}^x(\mathrm{a}\cos x + \mathrm{b}\sin x)$ is

Options:

A. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} - y = 0$

B. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 0$

C. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$

D. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 0$

Answer: D

Solution:

$y=e^x(a\cos x+b\sin x)$

$\Rightarrow \frac{dy}{dx}=e^x(a\cos x+b\sin x)+e^x(b\cos x-a\sin x)$

$\Rightarrow \frac{dy}{dx}=y+e^x(b\cos x-a\sin x)$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{dy}{dx}+e^x(b\cos x-a\sin x)$
$+e^x(-b\sin x-a\cos x)$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{dy}{dx}+\left(\frac{dy}{dx}-y\right)-y$ ... [From (i)]

$\Rightarrow \frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

Question 39

If $\int \frac{x^3\,dx}{\sqrt{1+x^2}}=a\left(1+x^2\right)\sqrt{1+x^2}+b\sqrt{1+x^2}+c$ (where c is a constant of integration), then the value of $3ab$ is

Options:

A. -3

B. -1

C. 1

D. 3

Answer: B

Solution:

Put $1+x^2=t^2$
$\Rightarrow 2x\,dx=2tdt$
$\Rightarrow x\,dx=tdt$

$$\therefore \quad \int \frac{x^3\,dx}{\sqrt{1+x^2}}$$

$$=\int \frac{t^2-1}{t}\cdot tdt$$

$$=\int (t^2-1)dt$$

$$=\frac{t^3}{3}-t+c$$

$$=\frac{(1+x^2)^{\frac{3}{2}}}{3}-\sqrt{1+x^2}+c$$

$$=\frac{1}{3}(1+x^2)\sqrt{1+x^2}-\sqrt{1+x^2}+c$$

$$\therefore \quad a=\frac{1}{3}, b=-1$$

$\Rightarrow 3ab=-1$

Question 40

The perpendiculars are drawn to lines L_1 and L_2 from the origin making an angle $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ respectively with positive direction of X-axis. If both the lines are at unit distance from the origin, then their joint equation is

Options:

A. $x^2-y^2+2\sqrt{2}y+2=0$

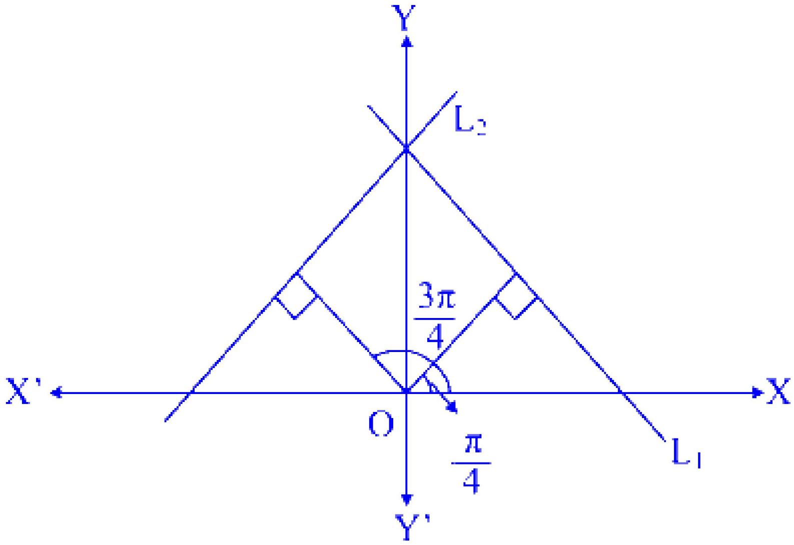
B. $x^2-y^2-2\sqrt{2}y-2=0$

C. $x^2-y^2+2\sqrt{2}y-2=0$

D. $x^2-y^2-2\sqrt{2}y+2=0$

Answer: C

Solution:



Equation of line L_1 is

$$x \cos \frac{\pi}{4} + y \sin \frac{\pi}{4} = 1$$
$$\Rightarrow \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 1$$
$$\Rightarrow x + y - \sqrt{2} = 0$$

Equation of line L_2 is

$$x \cos \frac{3\pi}{4} + y \sin \frac{3\pi}{4} = 1$$
$$\Rightarrow \frac{-x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 1$$
$$\Rightarrow x - y + \sqrt{2} = 0$$

$\therefore$ The joint equation of the lines is

$$(x + y - \sqrt{2})(x - y + \sqrt{2}) = 0$$
$$\Rightarrow x^2 - y^2 + 2\sqrt{2}y - 2 = 0$$

Question 41

The function $f(x) = [x] \cdot \cos\left(\frac{2x-1}{2}\right)\pi$, where $[\cdot]$ denotes the greatest integer function, is discontinuous at

Options:

A. all irrational numbers x.

B. no x.

C. all integer points.

D. every rational x which is not an integer.

Answer: C

Solution:

Greatest integer function is discontinuous on integer values.

$\therefore$ $\mathrm{f}(x)$ is discontinuous at all integer points.

Question 42

A line with positive direction cosines passes through the point $\mathrm{P}(2,-1,2)$ and makes equal angles with the co-ordinate axes. The line meets the plane $2x+y+z=9$ at point Q. The length of the line segment PQ equals

Options:

A. 3

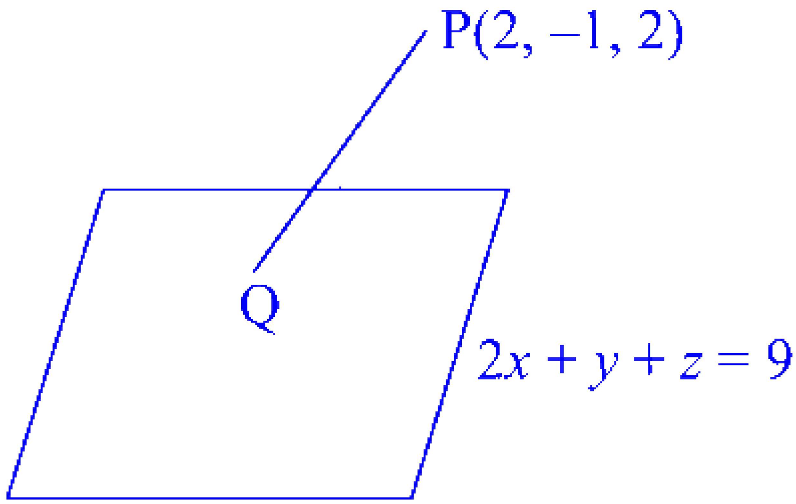
B. $\sqrt{2}$

C. $\sqrt{3}$

D. 2

Answer: C

Solution:



Since direction cosines of PQ are equal and positive.

$\therefore$ The d.r.s. of PQ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$\therefore$ The equation of the line PQ is

$$\frac{x-2}{\frac{1}{\sqrt{3}}} = \frac{y+1}{\frac{1}{\sqrt{3}}} = \frac{z-2}{\frac{1}{\sqrt{3}}}$$

$\Rightarrow x - 2 = y + 1 = \mathrm{z} - 2 = \mathrm{k}$, say

$\therefore$ Co-ordinates of the point Q are $(\mathrm{k}+2, \mathrm{k}-1, \mathrm{k}+2)$

The point Q lies on the plane $2x + y + z = 9$

$\therefore \quad 2(k+2) + k - 1 + k + 2 = 9$

$\Rightarrow 4k + 5 = 9 \quad \Rightarrow k = 1$

$\therefore \quad \mathrm{Q} \equiv (3, 0, 3)$

$\therefore \quad \mathrm{PQ} = \sqrt{(3-2)^2 + (0+1)^2 + (3-2)^2}$

$= \sqrt{1+1+1} = \sqrt{3}$

Question 43

If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{\lambda}$ and $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then sum of possible values of λ is

Options:

A. 16

B. 11

C. 12

D. 15

Answer: A

Solution:

The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-5}{5}$ is

$$d=\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1 b_2-a_2 b_1)^2+(a_1c_2-a_2c_1)^2+(b_1c_2-b_2c_1)^2}}\right|$$

$$\therefore \quad d=\left|\frac{\begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & \lambda \\ 1 & 4 & 5 \end{vmatrix}}{\sqrt{(8-3)^2+(10-\lambda)^2+(15-4\lambda)^2}}\right|$$

$$\begin{aligned}
&\Rightarrow \frac{1}{\sqrt{3}}=\left|\frac{5-2\lambda}{\sqrt{17\lambda^2-140\lambda+350}}\right| \\
&\Rightarrow \frac{1}{3}=\frac{25-20\lambda+4\lambda^2}{17\lambda^2-140\lambda+350} \\
&\Rightarrow 5\lambda^2-80\lambda+275=0 \\
&\Rightarrow \lambda^2-16\lambda+55=0 \\
&\Rightarrow (\lambda-5)(\lambda-11)=0 \\
&\Rightarrow \lambda=5 \text{ or } \lambda=11
\end{aligned}$$

$\therefore$ Sum of possible values of $\lambda=16$

Question 44

If the angles A, B, and C of a triangle are in an Arithmetic Progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A$ is

Options:

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{2}$

C. 1

D. $\sqrt{3}$

Answer: D

Solution:

A, B, C are in A.P.

$$\begin{aligned} \therefore \quad & \mathrm{A}+\mathrm{C}=2\,\mathrm{B} \\ & \text{Also, } \mathrm{A}+\mathrm{B}+\mathrm{C}=180^{\circ} \\ \therefore \quad & \mathrm{B}=60^{\circ} \end{aligned}$$

By sine rule,

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=k$$

$$\therefore \quad \sin A=ak, \sin B=bk, \sin C=ck$$

$$\therefore \quad \frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A$$

$$\begin{aligned} \therefore \quad & \frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A \\ & =\frac{a}{c}(2\sin C\cos C)+\frac{c}{a}(2\sin A\cos A) \\ & =\frac{a}{c}(2ck\cos C)+\frac{c}{a}(2ak\cos A) \\ & =2ka\cos C+2kc\cos A \\ & =2k(a\cos C+c\cos A) \end{aligned}$$

$$=2\,\mathrm{kb} \quad \ldots[\because \mathrm{b}=\mathrm{a}\cos\mathrm{C}+\mathrm{c}\cos\mathrm{A}]$$

$$\begin{aligned} & =2\sin\mathrm{B} \\ & =2\times\frac{\sqrt{3}}{2} \quad \ldots.[\because \angle\mathrm{B}=60^{\circ}] \\ & =\sqrt{3} \end{aligned}$$

Question 45

A linguistic club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this group including the selection of a leader (from among these 4 members) for the team. If the team has to include at most one boy, the number of ways of selecting the team is

Options:

A. 140

B. 320

C. 76

D. 380

Answer: D

Solution:

Case I : No boy is included.

Selecting 4 girls from 6 girls $= {}^{6}C_{4}$

Selecting 1 captain from selected members $= {}^{4}C_{1}$

Total number of ways $= {}^{6}C_{4} \times {}^{4}C_{1} = 60$

Case II: One boy is included.

Selecting 3 girls and 1 boy from given members

$= {}^{6}C_{3} \times {}^{4}C_{1}$.

Selecting 1 captain from the selected members $= {}^{4}C_{1}$.

Total Number of ways $= {}^{6}C_{3} \times {}^{4}C_{1} \times {}^{4}C_{1} = 320$.

$\therefore$ Total Number of ways $= 320 + 60 = 380$.

Question 46

The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $\mathrm{S} = \{x \in \mathrm{R} / x^2 + 30 \leq 11x\}$ is

Options:

A. 122

B. -122

C. -222

D. 222

Answer: A

Solution:

$$\begin{aligned} \mathrm{S} &= \{x \in \mathrm{R} / x^2 + 30 \leq 11x\} \\ &= \{x \in \mathrm{R} / x^2 - 11x + 30 \leq 0\} \\ &= \{x \in \mathrm{R} / (x-5)(x-6) \leq 0\} \\ &= \{x \in \mathrm{R} / x \in [5,6]\} \end{aligned}$$

$$\begin{aligned} \mathrm{f}(x) &= 3x^3 - 18x^2 + 27x - 40 \\ \therefore \quad \mathrm{f}'(x) &= 9x^2 - 36x + 27 \\ &= 9(x-1)(x-3) > 0 \quad \forall x \in [5,6] \end{aligned}$$

$\Rightarrow \mathrm{f}(x)$ is increasing in $[5,6]$.

$$\begin{aligned} \therefore \quad \text{Maximum value } &= \mathrm{f}(6) \\ &= 3(6)^3 - 18(6)^2 + 27(6) - 40 \\ &= 122 \end{aligned}$$

Question 47

Let $f(x) = \int \frac{x^2-3x+2}{x^4+1}\, \mathrm{d}x$, then function decreases in the interval

Options:

A. $(-\infty, -2)$

B. $(-2, -1)$

C. $(1, 2)$

D. $(2, \infty)$

Answer: C

Solution:

$$f(x)=\int \frac{x^2-3x+2}{x^4+1}\,\mathrm{d}x$$
$$\Rightarrow \mathrm{f}'(x)=\frac{x^2-3x+2}{x^4+1}$$

For $\mathrm{f}(x)$ to be decreasing,

$$\mathrm{f}'(x)<0$$
$$\Rightarrow \frac{x^2-3x+2}{x^4+1}<0$$
$$\Rightarrow \frac{(x-1)(x-2)}{x^4+1}<0$$
$$\Rightarrow (x-1)(x-2)<0$$
$$\Rightarrow x\in(1,2)$$

Question 48

Three critics review a book. For the three critics the odds in favour of the book are $2:5, 3:4$ and $4:3$ respectively. The probability that the majority is in favour of the book, is given by

Options:

A. $\frac{183}{343}$

B. $\frac{160}{343}$

C. $\frac{209}{343}$

D. $\frac{134}{343}$

Answer: D

Solution:

The probability that the first critic favours the book is $\mathrm{P}(\mathrm{A})=\frac{2}{2+5}=\frac{2}{7}$

$\therefore \quad \mathrm{P}(\mathrm{A}') = 1 - \frac{2}{7} = \frac{5}{7}$

The probability that the second critic favours the book is $P(B) = \frac{3}{3+4} = \frac{3}{7}$

$\therefore \quad \mathrm{P}(\mathrm{B}') = 1 - \frac{3}{7} = \frac{4}{7}$

The probability that the third critic favours the book is $\mathrm{P}(\mathrm{C}) = \frac{4}{4+3} = \frac{4}{7}$

$\therefore \quad P(C') = 1 - \frac{4}{7} = \frac{3}{7}$

$\therefore$ Majority will be in favour of the book if at least two critics favour the book. Hence, the probability is

$$\begin{aligned}
&\mathrm{P}\left(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}'\right) + \mathrm{P}\left(\mathrm{A} \cap \mathrm{B}' \cap \mathrm{C}\right) \\
&\quad + \mathrm{P}\left(\mathrm{A}' \cap \mathrm{B} \cap \mathrm{C}\right) + \mathrm{P}(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}) \\
&= \mathrm{P}(\mathrm{A}) \cdot \mathrm{P}(\mathrm{B}) \cdot \mathrm{P}\left(\mathrm{C}'\right) + \mathrm{P}(\mathrm{A}) \cdot \mathrm{P}\left(\mathrm{B}'\right) \cdot \mathrm{P}(\mathrm{C}) \\
&\quad + \mathrm{P}\left(\mathrm{A}'\right) \cdot \mathrm{P}(\mathrm{B}) \cdot \mathrm{P}(\mathrm{C}) + \mathrm{P}(\mathrm{A}) \cdot \mathrm{P}(\mathrm{B}) \cdot \mathrm{P}(\mathrm{C})
\end{aligned}$$

$$\begin{aligned}
&= \frac{2}{7} \times \frac{3}{7} \times \frac{3}{7} + \frac{2}{7} \times \frac{4}{7} \times \frac{4}{7} + \frac{5}{7} \times \frac{3}{7} \times \frac{4}{7} + \frac{2}{7} \times \frac{3}{7} \times \frac{4}{7} \\
&= \frac{18}{343} + \frac{32}{343} + \frac{60}{343} + \frac{24}{343} \\
&= \frac{134}{343}
\end{aligned}$$

Question 49

Consider the lines $\mathrm{L}_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{\mathrm{z}+1}{2}$

$\mathrm{L}_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{\mathrm{z}-3}{3}$, then the unit vector perpendicular to both L_1 and L_2 is

Options:

A. $\frac{-\hat{\imath}+7\hat{\jmath}+5\hat{k}}{5\sqrt{3}}$

B. $\frac{-\hat{\imath}-7\hat{\jmath}+5\hat{k}}{5\sqrt{3}}$

C. $\frac{+\hat{\imath}-7\hat{\jmath}+5\hat{k}}{5\sqrt{3}}$

D. $\frac{\hat{\imath}+7\hat{\jmath}+5\hat{k}}{5\sqrt{3}}$

Answer: B

Solution:

Lines L_1 and L_2 are parallel to the vectors $\bar{b}_1 = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\bar{b}_2 = \hat{i} + 2\hat{j} + 3\hat{k}$ respectively.

$\therefore$ The unit vector perpendicular to both L_1 and L_2 is $\hat{n} = \frac{\bar{b}_1 \times \bar{b}_2}{|\bar{b}_1 \times \bar{b}_2|}$

Now, $\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{i} - 7\hat{j} + 5\hat{k}$

$\therefore \quad \hat{n} = \frac{1}{5\sqrt{3}}(-\hat{i} - 7\hat{j} + 5\hat{k})$

Question 50

The area bounded by the curves $y = (x-1)^2, y = (x+1)^2$ and $y = \frac{1}{4}$ is

Options:

A. $\frac{1}{3}$ sq. units.

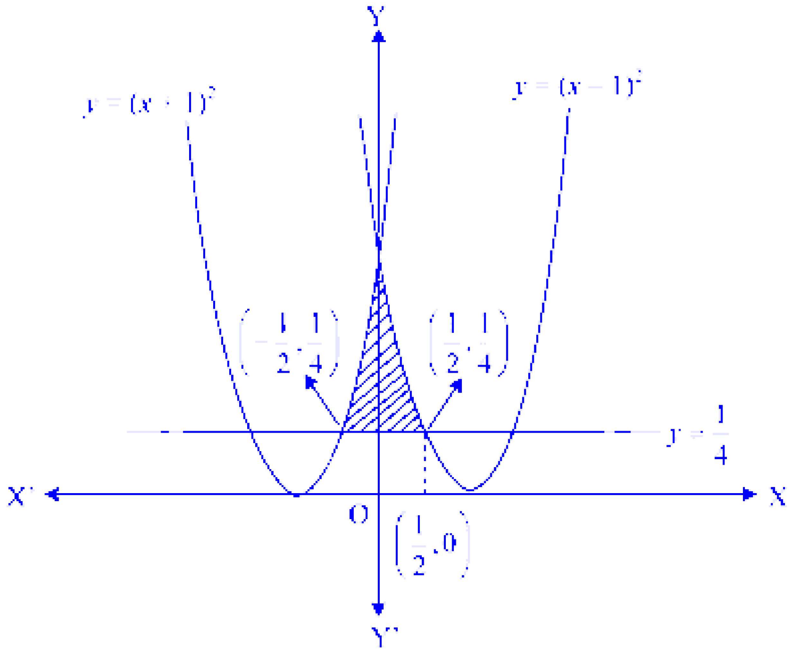
B. $\frac{2}{3}$ sq. units.

C. $\frac{1}{4}$ sq. units.

D. $\frac{1}{5}$ sq. units.

Answer: A

Solution:



$$\text{Required area } = 2\int_{0}^{\frac{1}{2}}\left[(x-1)^2-\frac{1}{4}\right]\mathrm{d}x$$

$$=2\left[\frac{(x-1)^3}{3}\right]_0^{\frac{1}{2}}-\frac{1}{2}[x]_0^{\frac{1}{2}}$$

$$=\frac{2}{3}\left(-\frac{1}{8}+1\right)-\frac{1}{2}\left(\frac{1}{2}-0\right)$$

$$=\frac{1}{3}\text{ sq. units}$$

Question 51

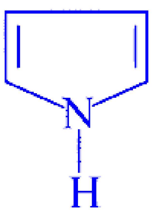
Which from following molecules does NOT contain nitrogen in it?

Options:

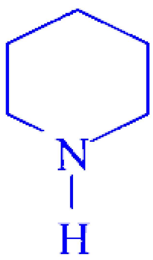
A. Pyrrole
B. Piperidine
C. Pyridine
D. Pyran

Answer: D

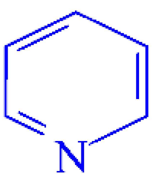
Solution:



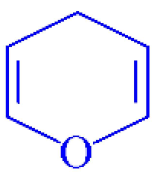
Pyrrole



Piperidine



Pyridine



Pyran

Question 52

Calculate the volume of unit cell if an element having molar mass 56 g mol^{-1} that forms bcc unit cells.

$$\left[\rho \cdot N_A = 4.8 \times 10^{24} \text{ g cm}^{-3} \text{ mol}^{-1}\right]$$

Options:

A. $1.17 \times 10^{-23} \text{ cm}^3$

B. $4.79 \times 10^{-23}\ \mathrm{cm}^3$

C. $3.31 \times 10^{-23}\ \mathrm{cm}^3$

D. $2.33 \times 10^{-23}\ \mathrm{cm}^3$

Answer: D

Solution:

We know that the density of the element is given by:

$\rho = \frac{ZM}{N_A V}$

Where:

* ρ is the density of the element

* Z is the number of atoms per unit cell (for bcc, Z = 2)

* M is the molar mass of the element

* N_A is Avogadro's number $6.022 \times 10^{23}\mathrm{mol}^{-1}$,

* V is the volume of the unit cell

We are given:

* $\rho N_A = 4.8 \times 10^{24}\ \mathrm{g\ cm}^{-3}\ \mathrm{mol}^{-1}$

* $M = 56\ \mathrm{g\ mol}^{-1}$

Substituting these values into the equation above, we get:

$V = \frac{ZM}{\rho N_A} = \frac{2 \times 56\ \mathrm{g\ mol}^{-1}}{4.8 \times 10^{24}\ \mathrm{g\ cm}^{-3}\ \mathrm{mol}^{-1}} = 2.33 \times 10^{-23}\ \mathrm{cm}^3$

Therefore, the volume of the unit cell is $2.33 \times 10^{-23}\ \mathrm{cm}^3$. So the correct answer is **Option D**.

Question 53

Find the number of orbitals and maximum electrons respectively present in M-shell?

Options:

A. $4, 8$

B. 9, 18

C. 16, 32

D. 1, 2

Answer: B

Solution:

Symbol of Shell	Value of Principal quantum number (n)	Value of Azimuthal Quantum number (l)	Symbol of subshell	Total Number of orbitals in the subshell $= 2l + 1$
M	$n = 3$	$l = 0$	3s	$2 \times 0 + 1 = 1$
		$l = 1$	3p	$2 \times 1 + 1 = 3$
		$l = 2$	3d	$2 \times 2 + 1 = 5$

Hence, the total number of orbitals $= 1 + 3 + 5 = 9$. Each orbital accommodates 2 electrons. Therefore, total number of electrons in M-Shell is 18.

Question 54

Which from following expressions is used to find the cell potential of $Cd_{(s)} \left| Cd^{++}_{(aq)} \right| \left| Cu^{+}_{(aq)} \right| Cu_{(s)}$ cell at $25°C$?

Options:

A. $E_{cell} = E^{o}_{cell} - 0.0296 \log \frac{[Cd^{++}]}{[Cu^{++}]}$

B. $E_{cell} = E^{o}_{cell} + 0.0296 \log \frac{[Cd^{++}]}{[Cu^{++}]}$

C. $E_{cell} = E^{o}_{cell} - 0.0592 \log \frac{[Cu^{++}]}{[Cd^{++}]}$

D. $E_{cell} = E^{\circ}_{cell} + 0.0592 \log \frac{[Cu^{++}]}{[Cd^{++}]}$

Answer: A

Solution:

Cell reaction

$$Cd_{(s)} \longrightarrow Cd^{2+}_{(s)} + 2e \quad \text{(oxidation at anode)}$$

$$Cu^{2+}_{(aq)} + 2e \longrightarrow Cu_{(s)} \quad \text{(reduction at cathode)}$$

$$Cd_{(s)} + Cu^{2+}_{(aq)} \longrightarrow Cd^{2+}_{(aq)} + 2Cu_{(s)}$$

$$\text{(overall reaction)}$$

Now,

$$E_{cell} = E^{o}_{cell} - \frac{0.0592\ V}{n} \log_{10} \frac{[\ Product\ ]}{[\ Reactant\ ]}$$

$$= 0.02 - \frac{0.0592\ V}{2} \log_{10} \frac{[Cd^{++}]}{[Cu^{++}]}$$

$$= E^{o}_{cell} - 0.0296 \log \frac{[Cd^{++}]}{[Cu^{++}]}$$

Question 55

Which of the following is formed when propene is heated with bromine at high temperature?

Options:

A. 1,2-Dibromopropane

B. 1-Bromopropane

C. 2-Bromopropene

D. 3-Bromopropene

Answer: D

Solution:

When alkenes are heated with Br_2 or Cl_2 at high temperature, hydrogen atom of allylic carbon is substituted with halogen atom giving allyl halide.

$$CH_2 = CH - CH_3 + Br_2 \longrightarrow \underset{3-\text{Bromopropene}}{CH_2 = CH - CH_2Br} + HBr$$

Question 56

Identify the product 'B' in the following sequence of reactions.

$$CH_3Br \xrightarrow{KCN} A \xrightarrow{NaC_2H_3OH} B$$

Options:

A. Methyl cyanide

B. Ethylamine

C. Methylamine

D. Ethyl cyanide

Answer: B

Solution:

$$CH_3Br \overset{KCN}{\longrightarrow} CH_3CN \overset{Na/C_2H_5OH}{\longrightarrow} \underset{\text{Ethyla min e}}{CH_3 = CH_2 - NH_2}$$

Question 57

Identify 'A' in the following reaction.

$$\textbf{A+ Acetic anhydride} \xrightarrow{H^+} \textbf{Aspirin + Acetic acid}$$

Options:

A. Acrylic acid

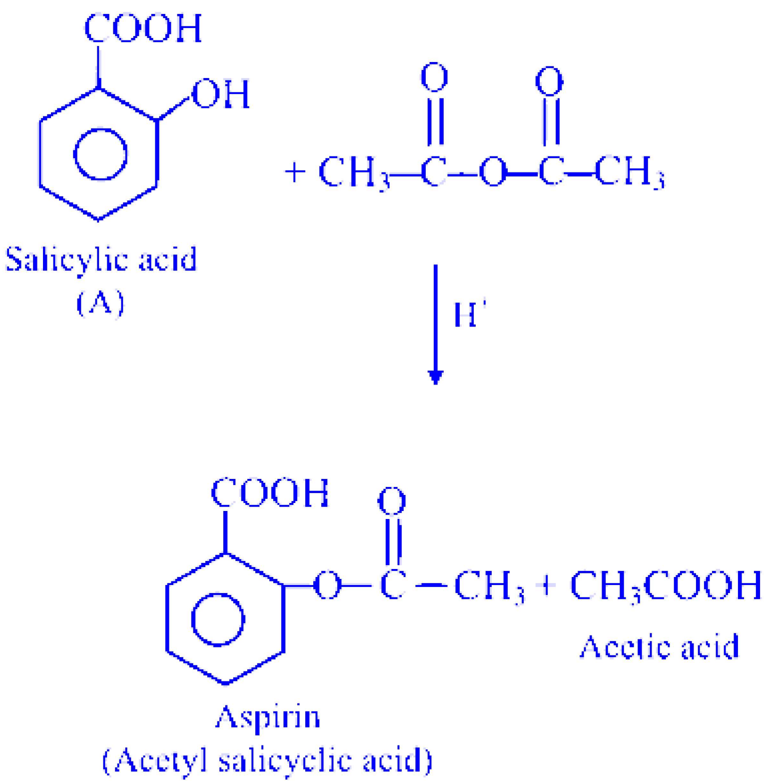
B. Oxalic acid

C. Salicylic acid

D. Phthalic acid

Answer: C

Solution:



Question 58

What is the value of $\angle S-S-S$ in puckered S_8 rhombic sulfur?

Options:

A. 107°

B. 120°

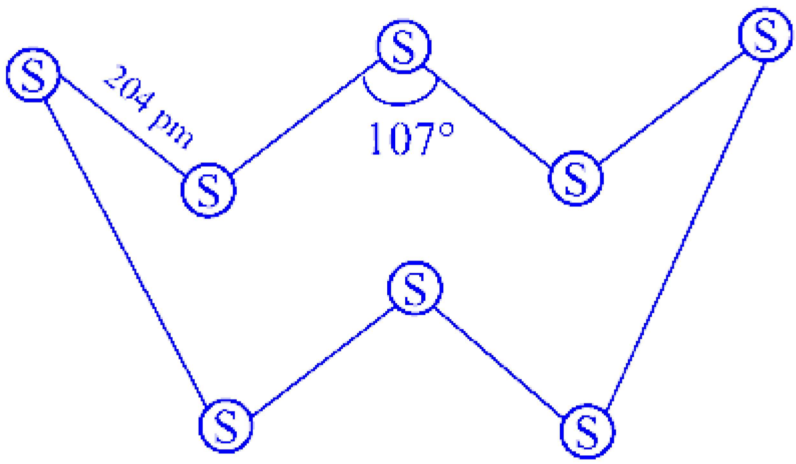
C. 104.5°

D. 60°

Answer: A

Solution:

Structure of S_8 ring in rhombic sulfur:



Question 59

Identify the reagent used in the following reaction.

Benzoic acid $\xrightarrow[\Delta]{\text{Reagent}}$ Benzoyl chloride + Phosphorous oxychloride + Hydrogen chloride

Options:

A. PCl_3

B. HCl

C. PCl_5

D. $SOCl_2$

Answer: C

Solution:

$$C_6H_5-COOH + PCl_5 \xrightarrow{\Delta} C_6H_5COCl + POCl_3 + HCl\uparrow$$

Benzoic acid → Benzoyol chloride + Phosphorous oxychloride + Hydrogen chloride

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Question 60

Which activity from following is exhibited by Lewis base according to definition?

Options:

A. accept a pair of electron

B. donate a pair of electron

C. accept H^+ ions

D. donate OH^- ions

Answer: B

Solution:

According to the definition given by G. N. Lewis, a Lewis base is a substance that can donate a pair of electrons to a Lewis acid to form a Lewis adduct. A Lewis base, therefore, is an electron pair donor. So, the correct answer to this question would be:

Option B: donate a pair of electrons.

Question 61

Which of the following ion has greater coagulating power for negatively charged sol?

Options:

A. $Fe(CN)_6^4$

B. PO_4^{3-}

C. Ba^{2+}

D. Al^{3+}

Answer: D

Solution:

In the coagulation of negative sol, greater the positive charge of the flocculating ion added, greater is its power to cause precipitation.

To precipitate negatively charged sol, positively charged ions are required.

Question 62

If enthalpy change for following reaction at $300 \mathrm{~K}$ is $+7 \mathrm{~kJ} \mathrm{~mol}^{-1}$ find the entropy change of surrounding?

$$H_2O_{(s)} \longrightarrow H_2O_{(\ell)}$$

Options:

A. $-42.8 \mathrm{~J} \mathrm{~K}^{-1}$

B. $-23.3 \mathrm{~J} \mathrm{~K}^{-1}$

C. $-30.7 \mathrm{~J} \mathrm{~K}^{-1}$

D. $-110.0 \mathrm{~J} \mathrm{~K}^{-1}$

Answer: B

Solution:

To find the entropy change of the surroundings for the given process, we can use the following thermodynamic relation which connects the enthalpy change (ΔH) of a system to the entropy change of the surroundings ($\Delta S_{\text{surroundings}}$) at constant temperature:

$$\Delta S_{\text{surroundings}} = -\frac{\Delta H}{T}$$

In this formula, ΔH is the enthalpy change for the system, and T is the temperature at which the process takes place. The negative sign indicates that if the process is endothermic ($\Delta H > 0$) for the system, the entropy of the surroundings decreases ($\Delta S_{\text{surroundings}} < 0$), and vice versa for an exothermic process.

For the reaction given:

$$H_2O_{(s)} \longrightarrow H_2O_{(\ell)}$$

the enthalpy change is given as $+7 \text{ kJ mol}^{-1}$ or $+7000 \text{ J mol}^{-1}$ when converted to joules, since $1 \text{ kJ} = 1000 \text{ J}$, and the temperature is 300 K. Plugging these values into our equation gives:

$$\Delta S_{\text{surroundings}} = -\frac{+7000 \text{ J mol}^{-1}}{300 \text{ K}}$$

To simplify this:

$$\Delta S_{\text{surroundings}} = -\frac{7000}{300} \text{ J K}^{-1} \text{ mol}^{-1}$$

$$\Delta S_{\text{surroundings}} = -23.333\ldots \text{ J K}^{-1} \text{ mol}^{-1}$$

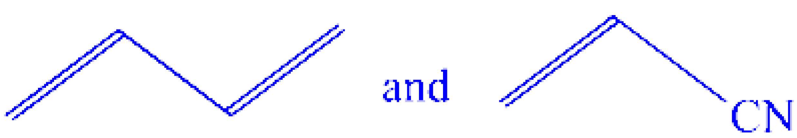
Therefore, the entropy change of the surroundings for the melting of ice at 300 K with an enthalpy change of $+7 \text{ kJ mol}^{-1}$ is approximately:

$$\Delta S_{\text{surroundings}} \approx -23.3 \text{ J K}^{-1} \text{ mol}^{-1}$$

This corresponds to Option B: -23.3 J K^{-1}.

Question 63

Identify the polymer obtained from



Options:

A. Polyacrylamide

B. Buna N

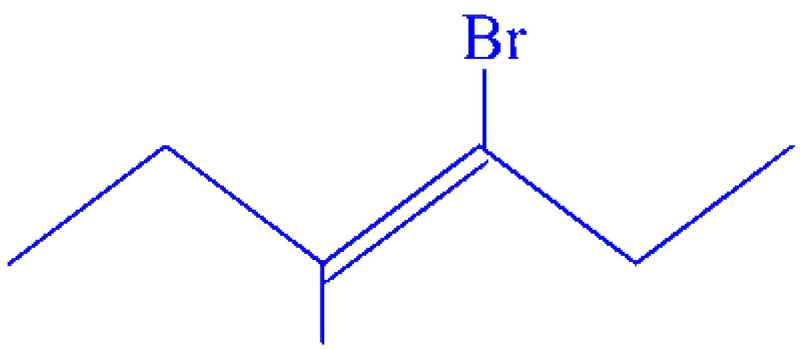
C. Glyptal

D. Perspex

Answer: B

Question 64

What is IUPAC name of the following compound?



Options:

A. 3-Bromo-4-ethylbut-3-ene

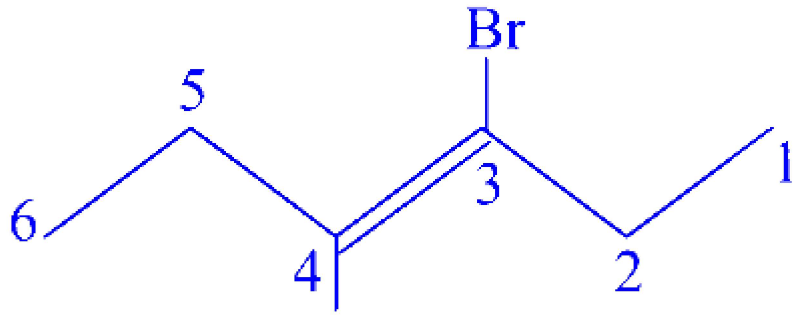
B. 3-Bromo-4-methylhex-3-ene

C. 4-Bromo-3-methylhex-3-ene

D. 4-Bromo-4-ethyl-3-methylbul-3-eue

Answer: B

Solution:



3-Bromo-4-methylhex-3-ene

Question 65

Calculate the pH of 0.01 M strong dibasic acid.

Options:

A. 5.5

B. 2.5

C. 2.0

D. 1.7

Answer: D

Solution:

$$\left[H_3O^+\right] = 2 \times c = 2 \times 0.01M = 2 \times 10^{-2}M$$
$$pH = -\log_{10}\left[H_3O^+\right]$$
$$= -\log_{10}\left[2 \times 10^{-2}\right]$$
$$= -\log_{10} 2 - \log_{10} 10^{-2}$$
$$= -\log_{10} 2 + 2$$
$$= 2 - 0.3010$$
$$pH = 1.7$$

When H_3O^+ concentration is $2 \times 10^{-2}M$, the pH should be less than 2. Only option (D) is valid.

Question 66

Which among the following cations produces colourless aqueous solution in their respective oxidation state?

Options:

A. Ti^{3+}

B. V^{3+}

C. Sc^{3+}

D. Cu^{2+}

Answer: C

Solution:

The color of the aqueous solutions of transition metal cations is generally due to electronic transitions among the d-orbitals as a consequence of ligand field splitting. Cations can absorb light of certain energies resulting in electronic transitions, which is what imparts color to their solutions.

Let's consider each of the given options:

Option A: Ti^{3+} (Titanium in +3 oxidation state) has one d electron ($3d^1$ configuration). This cation can undergo d-d transitions and is not colorless; it usually appears violet in aqueous solutions.

Option B: V^{3+} (Vanadium in +3 oxidation state) has two d electrons ($3d^2$ configuration). Like titanium, vanadium can also undergo d-d transitions and is typically green or blue in solutions, so it is not colorless either.

Option C: Sc^{3+} (Scandium in +3 oxidation state) has an electronic configuration of $3d^0$ after it has lost its three valence electrons. Since it has no d electrons, it cannot have d-d transitions and thus generally forms colorless solutions in water. Scandium is the correct answer to the question.

Option D: Cu^{2+} (Copper in +2 oxidation state) has one d electron ($3d^9$ configuration). It absorbs light in the red region, which makes its aqueous solution appear blue, so it is definitely not colorless.

Therefore, the cation that produces a colorless aqueous solution in its respective oxidation state is:

Option C: Sc^{3+}.

Question 67

What is the number of moles of electrons gained by one mole oxidizing agent in following redox reaction?

$Zn_{(s)} + 2HCl_{(aq)} \longrightarrow ZnCl_2 + H_2$

Options:

A. 4

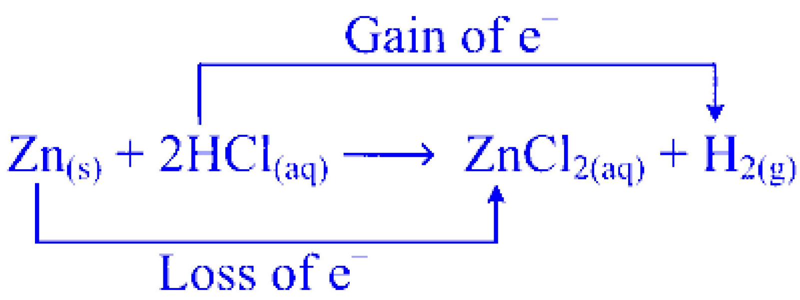
B. 3

C. 2

D. 1

Answer: D

Solution:



Oxidant/oxidising agent : HCl

2 moles of HCl gain 2 mol of electrons. Hence, 1 mole of HCl will gain 1 mole of electrons.

Question 68

Find the temperature in degree Celsius if volume and pressure of 2 mole ideal gas is $20\ dm^3$ and 4.926 atmospheres respectively. ($R = 0.0821\ dm^3\ atm\ K^{-1}\ mol^{-1}$)

Options:

A. 273

B. 327

C. 600

D. 453

Answer: B

Solution:

To find the temperature of an ideal gas, we can use the ideal gas equation:

$PV = nRT$

where:

- P is the pressure in atmospheres,
- V is the volume in liters (dm^3),
- n is the number of moles,
- R is the gas constant ($0.0821\ dm^3\ atm\ K^{-1}\ mol^{-1}$), and
- T is the temperature in Kelvin.

Given:

- $P = 4.926$ atmospheres,
- $V = 20\ dm^3 = 20$ L,
- $n = 2$ moles, and
- $R = 0.0821\ dm^3\ atm\ K^{-1}\ mol^{-1}$.

We can rearrange the ideal gas equation to solve for T:

$T = \frac{PV}{nR}$

Plugging in the given values:

$$T = \frac{(4.926\ \text{atm})(20\ \text{dm}^3)}{(2\ \text{mol})(0.0821\ \text{dm}^3\ \text{atm}\ \text{K}^{-1}\ \text{mol}^{-1})}$$

$$T = \frac{98.52\ \text{atm}\cdot\text{dm}^3}{0.1642\ \text{dm}^3\ \text{atm}\ \text{mol}^{-1}}$$

$T = 600$ K

To convert the temperature from Kelvin to Celsius, use the conversion formula:

Celsius = Kelvin − 273.15

For our calculation:

Celsius = 600 K − 273.15

Celsius = $326.85 \approx 327\ ^\circ$C

Therefore, the temperature of the gas in degree Celsius is closest to Option B: 327.

Question 69

What is the geometry of PCl_5 molecule as per VSEPR?

Options:

A. Trigonal bipyramidal

B. Octahedral

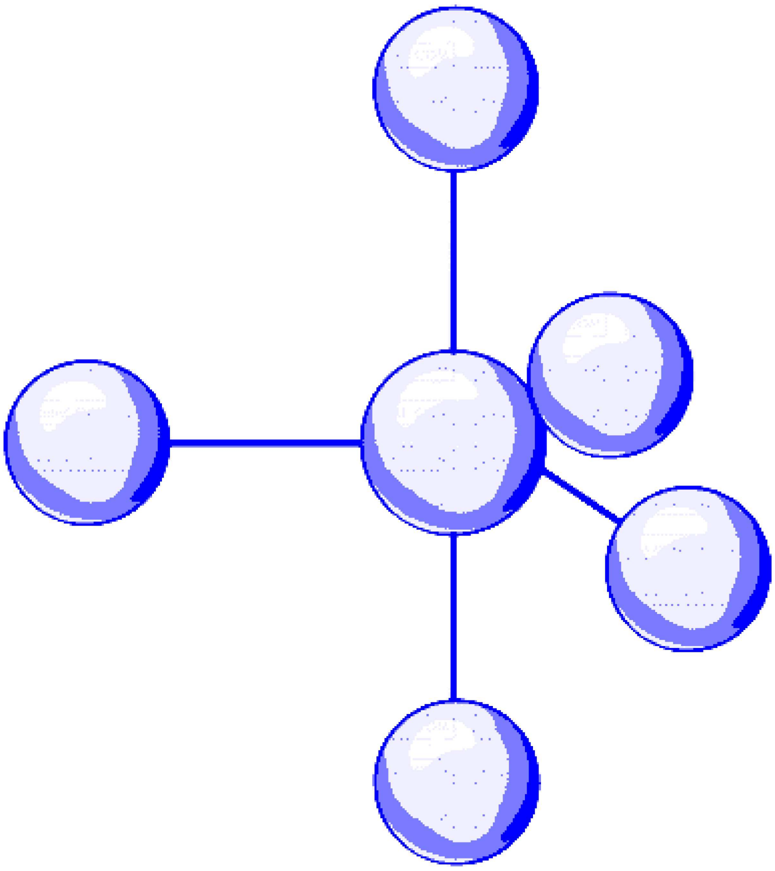
C. Tetrahedral

D. Square pyramidal

Answer: A

Solution:

PCl_5 : Trigonal bipyramidal



Question 70

What is coordination number of central metal ion in $[Fe(C_2O_4)_3]^{3-}$?

Options:

A. 3

B. 5

C. 4

D. 6

Answer: D

Solution:

In $[Fe(C_2O_4)_3]^{3-}$, three bidentate $C_2O_4^{2-}$ ligands, that is, total six donor atoms are attached to ferric ion (Fe^{3+}). Therefore, the coordination number of Fe^{3+} ion in the complex is six.

When coordination number (C.N.) of the metal ion in the complex is to be determined, always check the type of the ligands.

In case of monodentate ligands, C.N. $= 1\times$ (No. of monodentate ligands bound to the metal) and in case of bidentate ligands, C.N. $= 2\times$ (No. of bidentate ligands bound to the metal).

Question 71

Which among the following is NOT a true statement for enantiomers?

Options:

A. Enantiomers have identical refractive index.

B. Enantiomers are superimposable mirror images of each other.

C. Enantiomers have same chemical properties.

D. Enantiomers have equal but opposite optical ration.

Answer: B

Solution:

Enantiomers are non-superimposable mirror images of each other. This means that no matter how you rotate or flip one enantiomer, it will not perfectly align with its mirror image. Each enantiomer has a chiral center, or a carbon atom with four different substituents attached to it, leading to two different spatial arrangements that cannot be superimposed onto each other. Therefore, Option B, which states that "Enantiomers are superimposable mirror images of each other," is NOT a true statement about enantiomers.

As for the other options:

Option A is true: Enantiomers have the same physical properties, such as refractive index, boiling point, and melting point, in an achiral environment.

Option C is mostly true: Enantiomers have the same chemical properties in achiral environments or with achiral reagents, but they can behave differently in chiral environments or with chiral reagents, including biological systems.

Option D is true: Enantiomers have equal but opposite optical rotation; one enantiomer will rotate plane-polarized light to the right (dextrorotatory, or $+$) and the other to the left (levorotatory, or $-$) by the same magnitude, but in opposite directions.

Question 72

Which from following formulae is of sodium hexanitrocobaltate(III)?

Options:

A. $\mathrm{Na_3}\left[\mathrm{Co(NO_2)_6}\right]$

B. $\mathrm{Na_2}\left[\mathrm{Co(NO_2)_6}\right]$

C. $\left[\mathrm{Co(NaNO_2)_6}\right]$

D. $\mathrm{Na_3}\left[\mathrm{Co(ONO)_6}\right]$

Answer: A

Solution:

The oxidation state of metal ion is +3 . Hence, Sodium hexanitrocobaltate(III) : $\mathrm{Na_3}\left[\mathrm{Co(NO_2)_6}\right]$

Question 73

Which isomer among the following has the highest boiling point?

Options:

A. n-Butylaminc

B. tert-Butylamine

C. Ehyldimethylamine

D. Diethylamine

Answer: A

Solution:

In isomeric amines, boiling point decreases with increase in branching.

Question 74

Which element from following exhibits the highest number of allotropes?

Options:

A. O

B. S

C. Se

D. Te

Answer: B

Solution:

The element sulfur (S), option B, exhibits the highest number of allotropes among the options provided. Allotropy is the existence of two or more different forms of an element in the same physical state. Allotropes differ in the structure of the atoms and the type of bonding between them, resulting in different physical and chemical properties.

Sulfur is known to have several allotropes, with the most common one being rhombic sulfur (also known as α-sulfur), which consists of S_8 rings and is stable at room temperature. Another common allotrope is monoclinic sulfur (or β-sulfur), which also consists of S_8 rings but forms at temperatures above 95.6 degrees Celsius. Beyond these, sulfur can form several other polymeric forms, including various chain lengths and rings with different numbers of sulfur atoms.

In comparison, oxygen (O), selenium (Se), and tellurium (Te) exhibit fewer allotropes. For example:

- Oxygen primarily exists as O_2 (dioxygen) and O_3 (ozone).
- Selenium has a few allotropes, including red selenium (with chain-like structures) and gray selenium (with helical structures similar to S_8 but with more atoms per ring).
- Tellurium typically does not exhibit allotropy in the same way as the lighter chalcogens.

Therefore, sulfur (S), with its multiple allotropes, is the element that exhibits the highest number of allotropes among the options provided. Thus, Option B is the correct answer.

Question 75

Which of the following is a pair of dihydric phenols?

Options:

A. Resorcinol and Pyrogallol

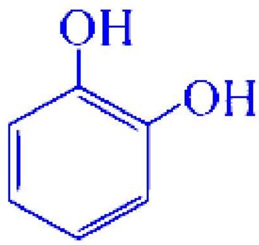
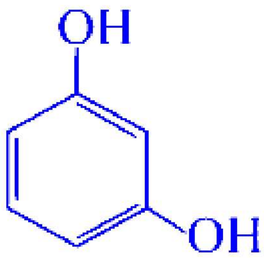
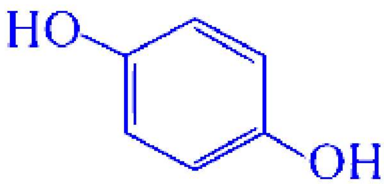
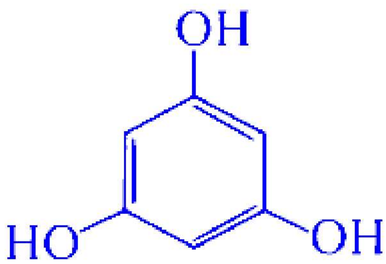
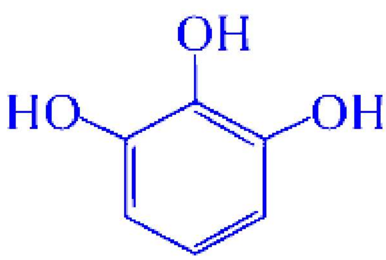
B. Quinol and Phloroglucinol

C. Phloroglucinol and Pyrogallol

D. Catechol and Quinol

Answer: D

Solution:

Structural formula	Common name
	Catechol
	Resorcinol
	Quinol
	Phloroglucinol
	Pyrogallol

Question 76

Calculate ΔH for following reaction, at $25^{\circ}C$.

$$NH_2CN_{(g)} + \frac{3}{2}O_{2(g)} \longrightarrow N_{2(g)} + CO_{2(g)} + H_2O_{(g)}$$

$$(\Delta U = -740.5 \text{ kJ}, R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1})$$

Options:

A. $-708.4 \text{ kJ mol}^{-1}$

B. $-789.4 \text{ kJ mol}^{-1}$

C. $-741.7 \text{ kJ mol}^{-1}$

D. $-863.9 \text{ kJ mol}^{-1}$

Answer: C

Solution:

$$\Delta n_g = 2 - \frac{3}{2} = \frac{1}{2} \text{ mol}$$
$$R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$$
$$= 8.314 \times 10^{-3} \text{ kJ K}^{-1} \text{ mol}^{-1}$$

Now, using formula,

$$\Delta H = \Delta U + \Delta n_g RT$$
$$= -740.5 \text{ kJ} + \left(\frac{1}{2} \text{ mol}\right) \times 8.314 \times 10^{-3} \text{ kJ K}^{-1} \text{ mol}^{-1} \times 298 \text{ K}$$

$$= -740.5 \text{ kJ} + 1.2388 \text{ kJ}$$
$$= -739.26 \text{ kJ mol}^{-1}$$
$$\approx -741.7 \text{ kJ mol}^{-1}$$

Question 77

What is number of atoms present in 2.24 dm^3 $NH_{3(g)}$ at STP?

Options:

A. 6.022×10^{22}

B. 2.4088×10^{23}

C. 1.8066×10^{22}

D. 6.022×10^{23}

Answer: B

Solution:

To determine the number of atoms present in $2.24\ dm^3$ of $NH_{3(g)}$ at standard temperature and pressure (STP), we should first recall the concept of molar volume. At STP, one mole of any ideal gas occupies $22.4\ dm^3$. We can use this information to calculate the number of moles of ammonia gas in the given volume.

The number of moles (n) is given by: $n = \frac{\text{Volume}}{\text{Molar Volume}}$

For NH_3 at STP: $n = \frac{2.24\ dm^3}{22.4\ dm^3/mol} = 0.1\ mol$

Each molecule of ammonia (NH_3) contains 1 nitrogen atom and 3 hydrogen atoms, totalling 4 atoms. To find the total number of atoms in 0.1 mole of ammonia, we multiply the number of atoms in one molecule by Avogadro's number (approximately 6.022×10^{23} atoms/mol), which tells us the number of particles (atoms, molecules, ions, etc.) in one mole of a substance.

The total number of atoms in 0.1 mole of NH_3 is:
Total Number of Atoms $=$ moles $\times$ Avogadro's number $\times$ atoms per molecule
$= 0.1$ moles $\times 6.022 \times 10^{23}$ atoms/mol $\times$ 4 atoms/molecule

Now we calculate the total number of atoms: Total Number of Atoms $= 0.1 \times 6.022 \times 10^{23} \times 4$
$= 0.1 \times 4 \times 6.022 \times 10^{23} = 2.4088 \times 10^{22}$ atoms

Therefore, the correct answer is Option B: 2.4088×10^{23} atoms.

Question 78

What is the number of moles of tertiary carbon atoms in a molecule of isobutane?

Options:

A. One

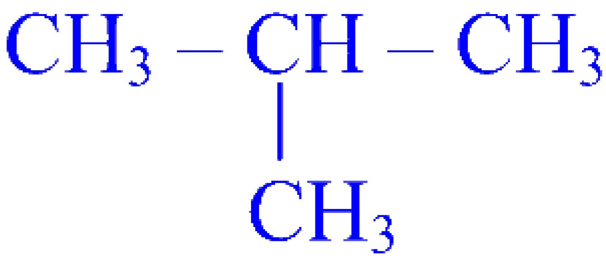
B. Two

C. Three

D. Four

Answer: A

Solution:



(One tertiary carbon atom)

Question 79

According to carbinol system, name of isopropyl alcohol is

Options:

A. Methyl carbinol

B. Ethyl carbinol

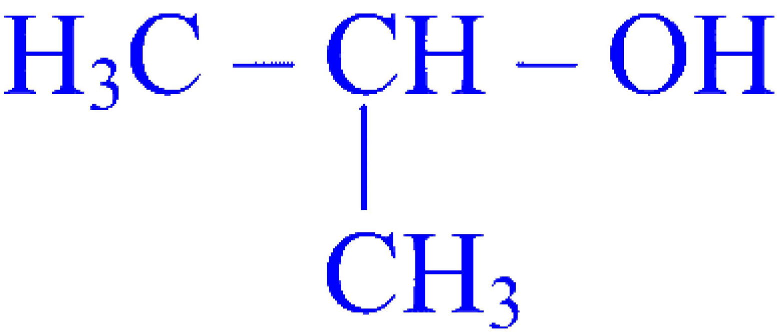
C. Dimethyl carbinol

D. Isopropyl carbinol

Answer: C

Solution:

In carbinol system, alcohols are considered as derivatives of methyl alcohol.



Dimethyl carbinol

Question 80

Calculate the relative lowering of vapour pressure if the vapour pressure of benzene and vapour pressure of solution of non-volatile solute in benzene are $640 \mathrm{~mmHg}$ and $590 \mathrm{~mmHg}$ respectively at same temperature.

Options:

A. 0.078

B. 0.175

C. 0.061

D. 0.092

Answer: A

Solution:

Relative lowering of vapour pressure

$$= \frac{\Delta P}{P_1^0} = \frac{P_1^0 - P_1}{P_1^0}$$

$$= \frac{640 - 590}{640} = 0.078$$

Question 81

Which from following is NOT true about voltaic cell?

Options:

A. The anode acts as negative electrode.

B. The cathode acts as a positive electrode.

C. It converts electrical energy into chemical energy

D. Dry cell is an example of voltaic cell.

Answer: C

Solution:

The correct answer is Option C: "It converts electrical energy into chemical energy." This statement is not true about a voltaic cell. Let's break down each of the given options to understand why Option C is incorrect:

Option A: **The anode acts as negative electrode**. This statement is true. In a voltaic cell, the anode is where oxidation occurs, meaning that it is the site where electrons are lost by the reactants. Because electrons are negatively charged, the anode accumulates negative charge relative to the cathode, so it acts as the negative electrode.

Option B: **The cathode acts as a positive electrode**. This statement is also true. In a voltaic cell, the cathode is where reduction occurs, meaning that it is the site where the electrons are gained by the reactants. Since the electrons come from the anode, the cathode acquires a positive charge relative to the anode, and hence it serves as the positive electrode.

Option C: **It converts electrical energy into chemical energy**. This statement is not true and is a characteristic of an electrolytic cell, not a voltaic cell. A voltaic cell, also known as a galvanic cell, operates on the principle of a spontaneous chemical reaction that produces electrical energy. It converts chemical energy into electrical energy, which can be used to do work.

Option D: **Dry cell is an example of a voltaic cell**. This statement is true. A dry cell is a common type of voltaic cell that is used in many household batteries. It generates electrical energy through spontaneous redox reactions occurring within the cell.

So, to reiterate, Option C is NOT true about a voltaic cell, as a voltaic cell indeed converts chemical energy into electrical energy, not the other way around.

Question 82

Calculate the percent atom economy when a product of formula weight 175u is obtained in a chemical reaction using 225u formula weight reactant.

Options:

A. 70.1%

B. 77.7%

C. 90.5%

D. 95.0%

Answer: B

Solution:

Atom economy is a measure of how efficiently a chemical reaction converts reactants into desired products. It is expressed as a percentage and is calculated using the formula:

$$\text{Atom Economy} = \left(\frac{\text{Molecular weight of the desired product}}{\text{Total molecular weight of reactants}}\right) \times 100\%$$

In the given problem, we have a product with a formula weight of 175u that is obtained from a reactant with a formula weight of 225u. Using the formula for atom economy, we can calculate:

$$\text{Atom Economy} = \left(\frac{175\text{u}}{225\text{u}}\right) \times 100\%$$

Now, divide 175 by 225 to find the value:

$$\text{Atom Economy} = \left(\frac{175}{225}\right) \times 100\% \text{ Atom Economy} = 0.7777\ldots \times 100\% \text{ Atom Economy} = 77.77\ldots\%$$

Rounding to one decimal place, the atom economy is approximately 77.8%. Among the options given, the closest answer is:

Option B 77.7%

Hence, option B is the correct answer.

Question 83

Identify false statement regarding isothermal process from following.

Options:

A. System can exchange heat energy with the surrounding.

B. Enthalpy of system remains constant.

C. Temperature of systems remains constant

D. Internal energy of system remint constant

Answer: B

Solution:

Let's examine each statement closely to identify the false one regarding an isothermal process:

Option A: System can exchange heat energy with the surrounding.

This statement is true. An isothermal process is one in which the temperature of the system remains constant. In order for the system to maintain constant temperature while performing work, there must be heat exchange with the surroundings.

Option B: Enthalpy of system remains constant.

This statement is false in general. Enthalpy, H, is a state function defined by $H = U + PV$, where U is the internal energy, P is the pressure, and V is the volume of the system. During an isothermal process, if the system does work or work is done on it (thus changing the volume in case the system is an ideal gas), then even if U remains constant (for an ideal gas), the product of $P \times V$ may change because the pressure can change. Therefore, the enthalpy H can indeed change during an isothermal process, especially for a process in which the pressure and volume change.

Option C: Temperature of systems remains constant.

This statement is true. By definition, an isothermal process occurs at a constant temperature.

Option D: Internal energy of system remains constant.

This statement can be true or false depending on the nature of the gas. Specifically, for an ideal gas, the internal energy depends only on the temperature. Therefore, during an isothermal process for an ideal gas, where the temperature is constant, the internal energy also remains constant. However, for real gases or other systems, internal energy could be a function of other variables as well, potentially leading to changes even in an isothermal process.

Based on the above analysis, the false statement regarding an isothermal process is **Option B**: Enthalpy of the system remains constant, as enthalpy can change depending on the pressure-volume work done on or by the system.

Question 84

Calculate dissociation constant of $0.001M$ weak monoacidic base undergoing 2% dissociation.

Options:

A. 4×10^{-7}

B. 2×10^{-6}

C. 2×10^{-7}

D. 1×10^{-7}

Answer: A

Solution:

$$\alpha = \frac{\text{Percent dissociation}}{100} = \frac{2}{100} = 0.02$$
$$K_a = \alpha^2 c = (0.02)^2 \times 0.001 = 4 \times 10^{-7}$$

Question 85

Find the rate law for the reaction, $CHCl_{3(g)} + Cl_{2(g)} \rightarrow CCl_{4(g)} + HCl_{(g)}$ if order of reaction with respect to $CHCl_{a(g)}$ is one and $\frac{1}{2}$ with $Cl_{2(g)}$.

Options:

A. $\text{Rate} = k\,[CHCl_3][Cl_2]^{1/2}$

B. $\text{Rate} = k[CHCl_3]^2[Cl_2]^{1/2}$

C. $\text{Rate} = k[CHCl_3]^{3/2}\,[Cl_2]$

D. $\text{Rate} = k[CHCl_3]^{1/2}\,[Cl_2]$

Answer: A

Solution:

The rate law for a reaction describes how the rate of the reaction depends on the concentration of the reactants. The rate law is determined experimentally and is generally given by the expression:

$\text{Rate} = k[\text{Reactant}_1]^m[\text{Reactant}_2]^n[\text{Reactant}_3]^p \ldots$

where k is the rate constant and m, n, p, etc., are the reaction orders with respect to each reactant.

For the given reaction,

$CHCl_{3(g)} + Cl_{2(g)} \rightarrow CCl_{4(g)} + HCl_{(g)}$,

the order of reaction with respect to $CHCl_{3(g)}$ is one, and the order with respect to $Cl_{2(g)}$ is $\frac{1}{2}$. Therefore, the rate law can be expressed as:

$\text{Rate} = k[CHCl_3]^1[Cl_2]^{\frac{1}{2}}$.

Comparing this expression to the given options, the correct rate law corresponds to:

Option A:
$\text{Rate} = \text{k}\,[CHCl_3][Cl_2]^{1/2}$

Question 86

The rate for reaction $2\text{ A} + \text{B} \rightarrow$ product is $6 \times 10^{-4}\text{ mol dm}^{-3}\text{ s}^{-1}$ Calculate the rate constant if the reaction is first order in A and zeroth order in B. [Given $[\text{A}] = [\text{B}] = 0.3\text{M}$]

Options:

A. $1 \times 10^{-3}\text{ s}^{-1}$

B. $2 \times 10^{-3}\text{ s}^{-1}$

C. $3 \times 10^{-3}\text{ s}^{-1}$

D. $4 \times 10^{-3}\text{ s}^{-1}$

Answer: B

Solution:

The rate law for a chemical reaction can be written based on the order of the reaction with respect to its reactants. For the given reaction:

$2A + B \rightarrow \text{product}$

and the information that it is first order in A (A) and zeroth order in B (B), the rate law is expressed as:

$\text{rate} = k[A]^1[B]^0$

Since the reaction is zeroth order in B, the concentration of B does not affect the rate and hence $[B]^0 = 1$. This simplifies the rate expression to:

$\text{rate} = k[A]^1$

The rate of the reaction is given as $6 \times 10^{-4}\ \text{mol dm}^{-3}\ \text{s}^{-1}$ and the concentrations of A and B are both 0.3M (M = mol/dm³). Substituting the rate and the concentration of A into the simplified rate law gives us:

$6 \times 10^{-4}\ \text{mol dm}^{-3}\ \text{s}^{-1} = k(0.3\text{M})^1$

Solving for the rate constant k:

$k = \frac{6\times10^{-4}\ \text{mol dm}^{-3}\ \text{s}^{-1}}{0.3\text{M}}$

Performing the division:

$k = \frac{6\times10^{-4}\ \text{s}^{-1}}{0.3}$ $k = 2 \times 10^{-3}\ \text{s}^{-1}$

Thus, the rate constant k for the reaction is $2 \times 10^{-3}\ \text{s}^{-1}$, which corresponds to Option B.

Question 87

Calculate molar conductivity of NH_4OH at infinite dilution if molar conductivities of $Ba(OH)_2$ $BaCl_2$ and NH_4Cl at infinite dilution are $520, 280, 129\Omega^{-1}\ \text{cm}^2\ \text{mol}^{-1}$ respectively.

Options:

A. $249.0\ \Omega^{-1}\ \text{cm}^2\ \text{mol}^{-1}$

B. $498.0\ \Omega^{-1}\ \text{cm}^2\ \text{mol}^{-1}$

C. $125.0\ \Omega^{-1}\ \text{cm}^2\ \text{mol}^{-1}$

D. $369.0\ \Omega^{-1}\ \text{cm}^2\ \text{mol}^{-1}$

Answer: A

Solution:

According to Kohlrausch law,

i. $\wedge_0\left(\mathrm{Ba}(\mathrm{OH})_2\right)=\lambda_{\mathrm{Ba}^{2+}}^0+2 \lambda_{\mathrm{OH}^{-}}^0$

ii. $\wedge_0\left(\mathrm{BaCl}_2\right)=\lambda_{\mathrm{Ba}^{2+}}^0+2 \lambda_{\mathrm{Cl}^{-}}^0$

iii. $\wedge_0\left(\mathrm{NH}_4 \mathrm{Cl}\right)=\lambda_{\mathrm{NH}_4{ }^{+}}^0+\lambda_{\mathrm{Cl}^{-}}^0$

Eq. (i) + $\frac{1}{2}$ Eq (ii) − $\frac{1}{2}$ Eq (iii) gives

$$\wedge_0\left(\mathrm{NH}_4 \mathrm{OH}\right)=\wedge_0\left(\mathrm{NH}_4 \mathrm{Cl}\right)+\frac{1}{2} \wedge_0\left(\mathrm{Ba}(\mathrm{OH})_2\right)-\frac{1}{2} \wedge_0\left(\mathrm{BaCl}_2\right)$$

$$\begin{aligned}\wedge_0\left(\mathrm{NH}_4 \mathrm{OH}\right) &=129+\frac{1}{2} 520-\frac{1}{2} 280 \\ &=249.0 \Omega^{-1} \mathrm{~cm}^2 \mathrm{~mol}^{-1}\end{aligned}$$

When using Kohlrausch law, remember to multiply molar ionic conductivities of cation and anion with the number of cations and anions, respectively, as in the chemical formula of the compound.

Question 88

Calculate radius of third orbit of He^{+}.

Options:

A. 52.9 pm

B. 105.8 pm

C. 238.1 pm

D. 423.2 pm

Answer: C

Solution:

$\mathrm{r}_{\mathrm{n}}=\frac{52.9(\mathrm{n})^2}{\mathrm{Z}} \mathrm{pm}$

He^{+} is a hydrogen-like species having $\mathrm{Z}=2$.

Radius of the third orbit of He^{+}

$= r_3 = \frac{52.9 \times (3)^2}{2} \text{pm} = 238.1 \text{ pm}$

Question 89

Calculate the depression in freezing point of solution when 4 g nonvolatile solute of molar mass 126 g mol^{-1} dissolved in 80 mL water [Cryoscopic constant of water $= 1.86 \text{ K kg mol}^{-1}$]

Options:

A. 0.55 K

B. 0.74 K

C. 0.86 K

D. 0.90 K

Answer: B

Solution:

To calculate the depression in freezing point, we use the formula derived from the concept of colligative properties:

$\Delta T_f = K_f \cdot \frac{w_2}{M_2} \cdot \frac{1}{w_1}$

Here:

- ΔT_f is the depression in freezing point.
- K_f is the cryoscopic constant of the solvent (water), which is $1.86 \text{ K kg mol}^{-1}$.
- w_2 is the mass of the solute, which is 4 g.
- M_2 is the molar mass of the solute, which is 126 g mol^{-1}.
- w_1 is the mass of the solvent, which we need to calculate from the given volume of water 80 mL.

First, we convert the volume of water to mass:

Since the density of water is approximately 1 g/mL,

$w_1 = 80 \text{ mL} \times 1 \text{ g/mL} = 80 \text{ g} = 0.08 \text{ kg}$

Now, substituting the values into the formula:

$\Delta T_f = 1.86 \cdot \frac{4}{126} \cdot \frac{1}{0.08}$

Simplify the expression step-by-step:

First calculate the fraction:

$\frac{4}{126} = 0.03175$

Then:

$\Delta T_f = 1.86 \cdot 0.03175 \cdot 12.5$

Next, we multiply 0.03175 by 12.5:

$0.03175 \cdot 12.5 = 0.396875$

Finally:

$\Delta T_f = 1.86 \cdot 0.396875 = 0.7381875 \approx 0.74 \text{ K}$

Therefore, the correct answer is:

Option B: 0.74 K

Question 90

In an ionic crystalline solid, atoms of element Y forms hcp structure. The atoms of element X occupy one third of tetrahedral voids. What is the formula of compound?

Options:

A. X_2Y_3

B. XY

C. X_3Y_3

D. XY_2

Answer: A

Solution:

To determine the formula of the compound, we need to understand the relationships between the number of atoms (or ions) in the hexagonal close-packed (hcp) structure and the number of tetrahedral voids available, along with how many of these voids are occupied by the atoms of element X.

In a hexagonal close-packed (hcp) structure, each atom of element Y contributes to the close-packed structure. For convenience, we can assume that there are 6 atoms of Y in the hcp arrangement. The reason for choosing 6 is because it's a multiple that simplifies calculations in the hcp lattice where each atom is surrounded by 12 others, but each atom itself is shared among multiple unit cells. Nonetheless, the exact number chosen for Y is irrelevant to the ratio we are trying to calculate, as the ratio of tetrahedral voids to atoms in an hcp structure remains constant regardless of the number of atoms considered.

In a close-packed structure, the number of tetrahedral voids is twice the number of atoms present. Thus, if we have 6 Y atoms in the hcp structure, we have $2 \times 6 = 12$ tetrahedral voids.

Given that atoms of element X occupy one-third of the tetrahedral voids, the number of X atoms occupying the tetrahedral voids is $\frac{1}{3} \times 12 = 4$.

Thus, we have 6 Y atoms and 4 X atoms. To find the simplest whole number ratio, we divide by the smallest number of atoms present among the elements, which gives us:

For Y: $\frac{6}{2} = 3$

For X: $\frac{4}{2} = 2$

This results in a formula of X_2Y_3, indicating that the correct answer is Option A.

Question 91

What is total number of crystal systems associated with 14 Bravais lattices?

Options:

A. 7

B. 14

C. 1

D. 3

Answer: A

Solution:

The total number of crystal systems associated with the 14 Bravais lattices is 7. These crystal systems are categorized based on the axes lengths and angles between them. Here is the list of crystal systems and the corresponding Bravais lattices:

1. **Cubic** (3 Lattices): Simple cubic, body-centered cubic, and face-centered cubic.

2. **Tetragonal** (2 Lattices): Simple tetragonal and body-centered tetragonal.
3. **Orthorhombic** (4 Lattices): Simple orthorhombic, base-centered orthorhombic, body-centered orthorhombic, and face-centered orthorhombic.
4. **Hexagonal** (1 Lattice): Simple hexagonal.
5. **Rhombohedral** (1 Lattice): Also known as trigonal.
6. **Monoclinic** (2 Lattices): Simple monoclinic and base-centered monoclinic.
7. **Triclinic** (1 Lattice): Simple triclinic.

Therefore, the correct answer is:

Option A: 7

Question 92

Which from following catalyst is used in decomposition of KCl_3 ?

Options:

A. Platinized asbestos

B. Fe-Cr catalyst

C. Ni

D. MnO_2

Answer: D

Solution:

The correct answer is Option D, MnO_2 (manganese dioxide).

Manganese dioxide (MnO_2) is commonly used as a catalyst in the decomposition of potassium chlorate (KClO_3) to release oxygen gas (O_2) and form potassium chloride (KCl). The reaction can be represented by the following chemical equation:

$$2\mathrm{KClO}_3 \xrightarrow{\mathrm{MnO}_2} 2\mathrm{KCl} + 3\mathrm{O}_2$$

Upon heating, the presence of MnO_2 lowers the activation energy of the decomposition reaction, thus enhancing the rate at which O_2 is produced without the catalyst itself being consumed in the reaction. This catalytic decomposition is of particular importance in applications where a ready source of O_2 is needed, such as in oxygen candles for submarines or aircraft, and in safety matches.

Question 93

Which from following polymers is used to obtain plastic dinner ware?

Options:

A. Backelite

B. Teflon

C. Melamine-formaldehyde

D. Polyacrylonitrile

Answer: C

Solution:

Out of the options provided for polymers used to obtain plastic dinnerware, the correct answer is:

Option C: Melamine-formaldehyde

Melamine-formaldehyde resin is a type of plastic known for its durability and heat resistance, making it particularly suitable for use in the manufacturing of dinnerware such as plates, bowls, and utensils. This kind of plastic does not easily get scratched and is also resistant to stains, which adds to its suitability for dinnerware that undergoes frequent use and washing.

Here's a brief overview of the options to understand why the others are not used for plastic dinnerware:

- **Option A: Bakelite -** Bakelite is a phenol-formaldehyde resin, which is known for being one of the first synthetic plastics. It is heat resistant and has been used for a variety of applications including electrical insulators and radio and telephone casings. However, it is not typically used for dinnerware.
- **Option B: Teflon -** Teflon is the brand name for a polymer known as polytetrafluoroethylene (PTFE). It is best known for its non-stick properties, which is why it is often used to coat cookware. Teflon itself isn't used to make dinnerware items like plates and cups.
- **Option D: Polyacrylonitrile -** Polyacrylonitrile is a synthetic, semicrystalline organic polymer resin, used primarily to make fibers. It is not commonly used in the production of dinnerware.

As such, melamine-formaldehyde resin is the correct choice for a polymer used in the manufacture of plastic dinnerware.

Question 94

Identify non reducing sugar from following.

Options:

A. Sucrose

B. Maltose

C. Lactose

D. Glucose

Answer: A

Solution:

To identify a non-reducing sugar from the options provided, we need to consider the structure and chemical reactivity of each. Reducing sugars are those that have a free aldehyde or ketone group capable of acting as a reducing agent, typically through an oxidation-reaction where the sugar is oxidized while reducing another compound. Non-reducing sugars, on the other hand, do not have such free reactive groups.

Below, I'll explain the structure of each sugar given in the options to identify the non-reducing sugar.

- **Sucrose:** Sucrose is a disaccharide composed of glucose and fructose units. It is unique because in its structure, the anomeric carbon of glucose (which is the carbon linked to both the oxygen in the ring and the -CH2OH group outside the ring) is involved in the glycosidic bond with the anomeric carbon of fructose. As a result, neither glucose nor fructose parts of sucrose have a free anomeric carbon capable of acting as a reducing agent. Since the defining groups that classify sugars as reducing are "locked away" in the glycosidic bond, sucrose is considered a non-reducing sugar.

- **Maltose:** Maltose is a disaccharide made up of two glucose units. In the structure of maltose, one of the glucose units has a free anomeric carbon that is not involved in the glycosidic bond. This free anomeric carbon allows maltose to act as a reducing sugar.

- **Lactose:** Lactose is a disaccharide composed of glucose and galactose units. Similar to maltose, lactose has a free anomeric carbon on the galactose part of the molecule, which is not involved in the glycosidic bond, thus permitting lactose to behave as a reducing sugar.

- **Glucose:** Glucose is a monosaccharide with an aldehyde group, allowing it to be oxidized and hence act as a reducing sugar. Therefore, glucose is indeed a reducing sugar.

Given this information, the correct answer is:

Option A - Sucrose, as it is the only non-reducing sugar in the list due to both anomeric carbons being involved in the glycosidic bond. Therefore, it cannot participate in redox reactions that are characteristic of reducing sugars.

Question 95

In which of the following carbohydrate, molecular mass increases by 84u after complete acetylation?

Options:

A. Aldotriose

B. Aldotetrose

C. Ketotetrose

D. Ketopentose

Answer: A

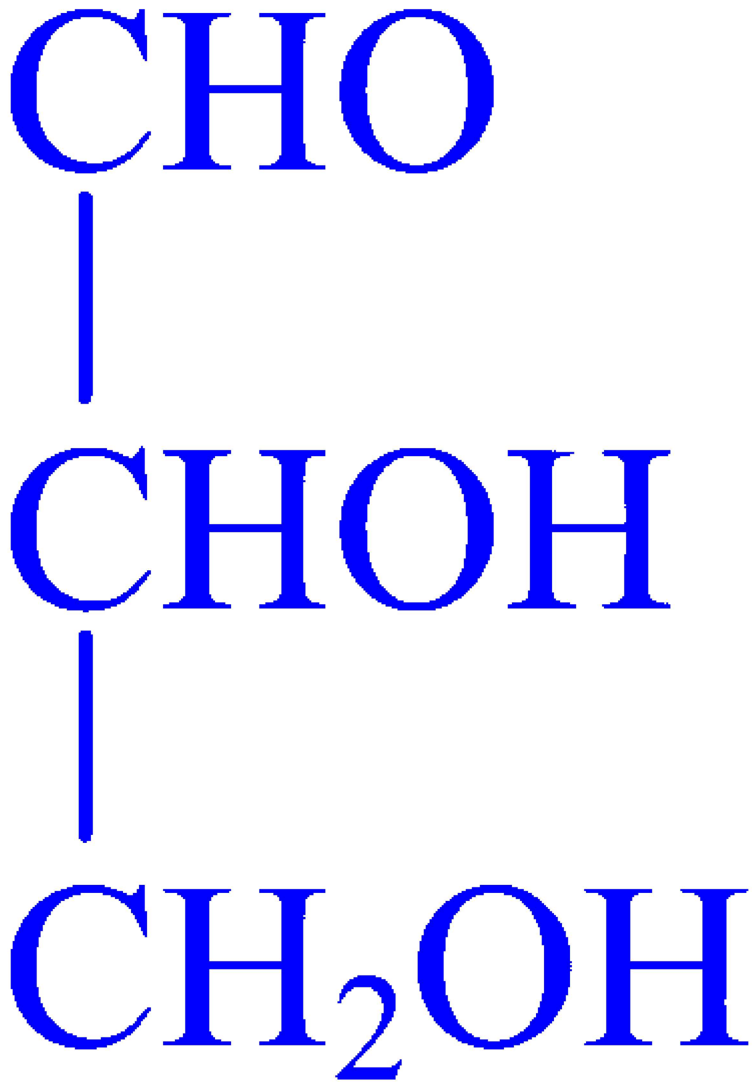
Solution:

In acetylation reaction H atom of an $(-OH)$ group is replaced by an acetyl group $(-COCH_3)$. This results increase in molecular mass by $[(12 + 16 + 12 + 3 \times 1) - 1]$, that as, 42u.

Increase in molecular mass $= 84u$

$\therefore$ Number of $-OH$ groups $= \frac{84u}{42u} = 2$

The carbohydrate must be aldotriose as it contains two alcoholic groups.



Question 96

Which element from following exhibits common oxidation state +2 ?

Options:

A. Sr

B. Rb

C. Na

D. Li

Answer: A

Solution:

The element from the options provided that commonly exhibits an oxidation state of +2 is Strontium (Sr). Let's elaborate on this answer.

Strontium belongs to the group 2 elements of the periodic table, which are also known as the alkaline earth metals. Elements in this group typically lose two electrons to form ions with a +2 charge in order to achieve a noble gas configuration. This means that Sr^{2+} is the common ion formed by strontium when it reacts with nonmetals or in other chemical reactions where it is oxidized.

Now, let's consider the other options:

- Rubidium (Rb) - This is an alkali metal that belongs to group 1 of the periodic table. Alkali metals typically have an oxidation state of +1 due to the loss of their single valence electron to form a cation (Rb^{+}).
- Sodium (Na) - Similar to rubidium, sodium is also an alkali metal and typically loses one electron to form an Na^{+} ion with a +1 oxidation state.
- Lithium (Li) - Lithium is the lightest metal and the first element in the alkali metals group. Like the other group 1 elements, it exhibits an oxidation state of +1 when it forms a Li^{+} ion.

In summary, Sr most commonly exhibits the +2 oxidation state, making option A (Sr) the correct answer to the question.

Question 97

Calculate half life of first order reaction if rate constant of reaction is $2.772 \times 10^{-3}\ s^{-1}$

Options:

A. 125 s

B. 250 s

C. 100 s

D. 150 s

Answer: B

Solution:

In the case of a first-order reaction, the half-life (which is the time taken for half of the reactant to be used up in the reaction) can be calculated using the following formula:

$t_{1/2} = \frac{0.693}{k}$

where $t_{1/2}$ is the half-life and k is the rate constant of the reaction.

Given that the rate constant $k = 2.772 \times 10^{-3}\ \mathrm{s}^{-1}$, we can plug this into our formula to find the half-life:

$t_{1/2} = \frac{0.693}{2.772 \times 10^{-3}\ \mathrm{s}^{-1}}$

To calculate this, first divide 0.693 by 2.772×10^{-3}:

$t_{1/2} = \frac{0.693}{2.772 \times 10^{-3}}$

$t_{1/2} = 250\ \mathrm{s}$

Therefore, the half-life of the reaction is 250 seconds. The correct option is:

Option B : 250 s

Question 98

Which among the following is NOT colligative property?

Options:

A. Vapour pressure lowering

B. Boiling point

C. Freezing point depression

D. Osmotic pressure

Answer: B

Solution:

Colligative properties are those properties of solutions that depend on the number of dissolved particles in solution, but not on their identity. Among the options given, all except Option B (Boiling point) refer directly to colligative properties.

Before clarifying why boiling point alone isn't the correct answer, let's define each term:

Vapour Pressure Lowering (Option A): A colligative property that refers to the decrease in the vapor pressure of a solvent when a non-volatile solute is dissolved in it compared to the vapor pressure of the pure solvent.

Freezing Point Depression (Option C): Another colligative property which states that the freezing point of a solvent will be lower when a solute is dissolved in it compared to the freezing point of the pure solvent.

Osmotic Pressure (Option D): Also a colligative property, osmotic pressure is the pressure required to stop the flow of solvent molecules through a semipermeable membrane from a region of low solute concentration to a region of high solute concentration.

Question 99

Identify the reaction in which carbonyl group of aldehydes and ketones is reduced to methylene group on treatment with hydrazine followed by heating with sodium hydroxide in ethylene glycol.

Options:

A. Wolf-Kishner reduction

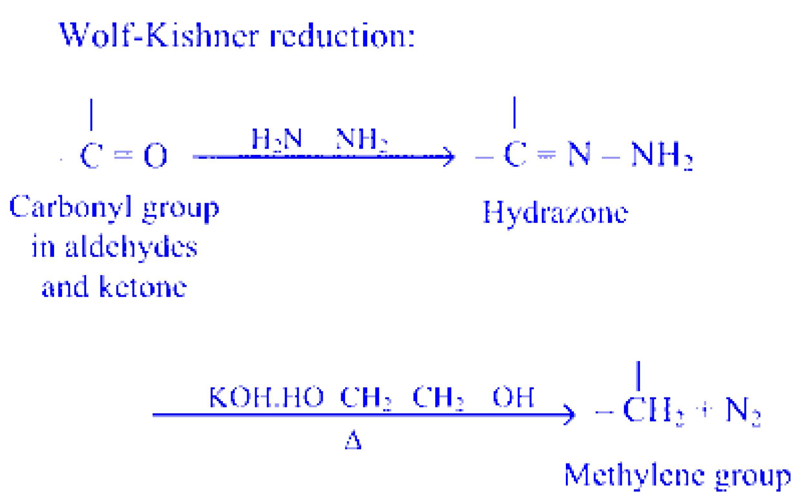
B. Clemmensen reduction.

C. Stephen reaction

D. Etard reaction

Answer: A

Solution:



Question 100

Identify the product 'B' in the following reaction. Toluene $\xrightarrow[CS_2]{\text{Chromylchloride}} A \xrightarrow{H_3O^+} B$

Options:

A. Benzal chloride

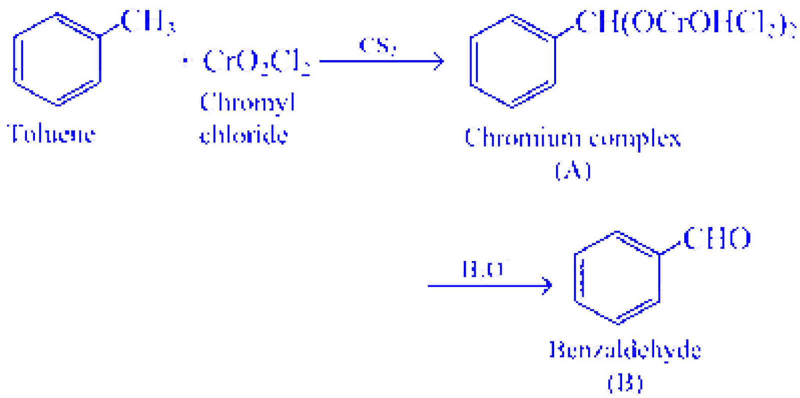
B. Benzaldehyde

C. Benzyl alcohol

D. Benzoic acid

Answer: B

Solution:



Physics

Question 101

A sphere and a cube, both of copper have equal volumes and are black. They are allowed to cool at same temperature and in same atmosphere. The ratio of their rate of loss of heat will be

Options:

A. 1 : 1

B. $\left(\frac{\pi}{6}\right)^{\frac{2}{3}}$

C. $\left(\frac{\pi}{6}\right)^{\frac{1}{3}}$

D. $\frac{4\pi}{3}$: 1

Answer: C

Solution:

Volumes of the cube and the sphere are equal

$$\therefore \quad a^3 = \frac{4}{3}\pi r^3$$
$$\therefore \quad a = \left(\frac{4}{3}\pi r^3\right)^{\frac{1}{3}}$$

Rate of loss of radiation is proportional to the surface area of the object.

$$\therefore \quad \frac{Q_{\text{sphere}}}{Q_{\text{cube}}} = \frac{4\pi r^2}{6\mathrm{a}^2}$$
$$\therefore \quad \frac{Q_{\text{sphere}}}{Q_{\text{cube}}} = \frac{4\pi \mathrm{r}^2}{6\left(\frac{4}{3}\pi \mathrm{r}^3\right)^{\frac{2}{3}}}$$
$$\therefore \quad \frac{Q_{\text{sphere}}}{Q_{\text{cube}}} = \left(\frac{\pi}{6}\right)^{\left(\frac{1}{3}\right)}$$

Question 102

An alternating voltage is applied to a series LCR circuit. If the current leads the voltage by 45°, then $(\tan 45^{\circ}=1)$

Options:

A. $\mathrm{X_L}=\mathrm{X_C}-\mathrm{R}$

B. $\mathrm{X_L}=\mathrm{X_C}+\mathrm{R}$

C. $\mathrm{X_C}=\sqrt{\mathrm{X_L^2}+\mathrm{R^2}}$

D. $\mathrm{X_L}=\sqrt{\mathrm{X_C^2}+\mathrm{R^2}}$

Answer: B

Solution:

The phase between the voltage and the current is given as

$$\tan \phi=\frac{X_L-X_C}{R}$$
$$\therefore \quad \tan 45^{\circ}=\frac{X_L-X_C}{R}$$
$$\therefore \quad 1=\frac{X_L-X_C}{R}$$
$$\therefore \quad R=X_L-X_C$$
$$\therefore \quad X_L=X_C+R$$

Question 103

A horizontal wire of mass 'm', length 'l' and resistance 'R' is sliding on the vertical rails on which uniform magnetic field 'B' is directed perpendicular. The terminal speed of the wire as it falls under the force of gravity is ($\mathrm{g}=$ acceleration due to gravity)

Options:

A. $\frac{\mathrm{mg}l}{\mathrm{BR}}$

B. $\frac{B^2 l^2}{mgR}$

C. $\frac{mgR}{Bl}$

D. $\frac{mgR}{B^2 l^2}$

Answer: D

Solution:

Net force on the wire becomes zero when it attains terminal velocity.

$\therefore$ Force due to magnetic field = gravitational force

$\therefore$ $iBl = mg$

$$\therefore \quad \frac{e}{R}Bl = mg \quad \ldots\left(\because i = \frac{e}{R}\right)$$

$$\therefore \quad \frac{Bvl}{R}Bl = mg \quad \ldots(\because e = Bvl)$$

$$\therefore \quad v = \frac{mgR}{B^2 l^2}$$

Question 104

Frequency of the series limit of Balmer series of hydrogen atom in terms of Rydberg's constant (R) and velocity of light (c) is

Options:

A. 4Rc

B. $\frac{4}{Rc}$

C. Rc

D. $\frac{Rc}{4}$

Answer: D

Solution:

Wavelength of Balmer series is given as, $\frac{1}{\lambda} = \mathrm{R}\left(\frac{1}{\mathrm{n}_1^2} - \frac{1}{\mathrm{n}_2^2}\right)$ where $\mathrm{n}_1 = 2$

For series limit, $\mathrm{n}_2 = \infty$

$\therefore \quad \frac{1}{\lambda} = \mathrm{R}\left(\frac{1}{4} - \frac{1}{\infty}\right) = \mathrm{R}\left(\frac{1}{4}\right)$

Now, $v = \frac{c}{\lambda}$

$\therefore \quad v = \frac{Rc}{4}$

Question 105

A string is stretched between two rigid supports separated by 75 cm. There are no resonant frequencies between 420 Hz and 315 Hz. The lowest resonant frequency for the string is

Options:

A. 210 Hz

B. 180 Hz

C. 105 Hz

D. 1050 Hz

Answer: C

Solution:

As there is no resonant frequency between 315 Hz and 420 Hz, let 315 Hz be n^{th} overtone and 420 Hz be $(\mathrm{n}+1)^{\text{th}}$ overtone.

Now, $v = \frac{\mathrm{nv}}{2l}$ ..... (i)

$\therefore \quad 315 = \frac{\mathrm{nv}}{2l}$ and $420 = \frac{(\mathrm{n}+1)\mathrm{v}}{2l}$

Taking the ratio,

$$\frac{315}{420} = \frac{n}{n+1}$$

$\therefore \quad 315n + 315 = 420n$

$\therefore \quad n = 3$

The resonant frequency is $\nu_0 = \frac{v}{2l}$

Therefore, from equation (i) we get,

$\nu_0 = \frac{\nu}{\mathrm{n}} = \frac{315}{3} = 105 \text{ Hz}$

Question 106

A straight wire carrying a current (I) is turned into a circular loop. If the magnitude of the magnetic moment associated with it is 'M', then the length of the wire will be

Options:

A. $\frac{\mathrm{M}\pi}{4\mathrm{I}}$

B. $\left[\frac{4\pi I}{M}\right]^{\frac{1}{2}}$

C. $\left[\frac{4\mathrm{M}\pi}{\mathrm{I}}\right]^{\frac{1}{2}}$

D. $4\pi\mathrm{MI}$

Answer: C

Solution:

Magnetic moment is given as $M = IA$

$\therefore \quad \mathrm{M} = \mathrm{I}\left(\pi\mathrm{R}^2\right)$

Now, length of the wire is, $L = 2\pi R$

$$\therefore \quad R = \frac{L}{2\pi}$$

$$\Rightarrow \quad M = I\pi\left(\frac{L}{2\pi}\right)^2$$

$$\therefore \quad L = \sqrt{\frac{4M\pi}{I}}$$

Question 107

The diffraction fringes obtained by a single slit are of

Options:

A. equal width

B. equal width and unequal intensity

C. unequal width but equal intensity

D. unequal width and unequal intensity

Answer: D

Solution:

When a wave encounters an obstacle, such as a slit, that is comparable in size to its wavelength, diffraction occurs. This phenomenon can result in the formation of a pattern of bright and dark regions called fringes on the other side of the slit. In the case of a single-slit diffraction, both the width and the intensity of the fringes vary.

The central maximum is the brightest and the widest. As one moves away from the center towards the edges, the intensity of the fringes decreases and their width becomes narrower. Thus, the width and the intensity of the fringes are not constant. The intensity falls off more slowly than the width, but both are functions of the angle from the central maximum.

The diffraction pattern for a single-slit can be explained using the Huygens-Fresnel principle, where each point on a wavefront within the slit is considered as a source of secondary spherical wavelets, and the wavelets interfere with each other to produce the diffraction pattern.

The width of the diffraction fringes can be described mathematically by the formula that gives the position of the minima: $d \sin\theta = m\lambda$ where: d is the width of the slit, θ is the diffraction angle, m is the order of the minimum (with $m = \pm1, \pm2, \pm3, \ldots$), and λ is the wavelength of the light.

As m increases, the value of $\sin\theta$ for the minima also increases, but not linearly, and thus the spacing between fringes changes. The intensity distribution of the single-slit diffraction pattern is governed by the intensity function, which shows that the intensity of the fringes falls off as a function of the angle θ.

The correct answer is : Option D - unequal width and unequal intensity.

Question 108

A particle moves around a circular path of radius 'r' with uniform speed 'V'. After moving half the circle, the average acceleration of the particle is

Options:

A. $\frac{\mathrm{V}^2}{\mathrm{r}}$

B. $\frac{2V^2}{r}$

C. $\frac{2V^2}{\pi r}$

D. $\frac{\mathrm{V}^2}{\pi \mathrm{r}}$

Answer: C

Solution:

At end points of the half revolution magnitude of the velocity is same but it directs in opposite direction.

$\therefore \quad \Delta \mathrm{V} = \mathrm{V} - (-\mathrm{V})$
$\therefore \quad \Delta \mathrm{V} = 2\ \mathrm{V}$

Time taken to complete the half revolution is

$\mathrm{t} = \frac{\pi \mathrm{r}}{\mathrm{V}}$

Average acceleration is, $a = \frac{\Delta V}{t} = \frac{2V}{\frac{\pi r}{V}}$

$\therefore \quad a = \frac{2V^2}{\pi r}$

Question 109

On dry road, the maximum speed of a vehicle along a circular path is 'V'. When the road becomes wet, maximum speed becomes $\frac{\mathrm{V}}{2}$. If coefficient of friction of dry road is 'μ' then that of wet road is

Options:

A. $\frac{2\mu}{3}$

B. $\frac{\mu}{4}$

C. $\frac{\mu}{3}$

D. $\frac{3\mu}{4}$

Answer: B

Solution:

The equation for maximum velocity is

$V = \sqrt{\mu rg}$ .... (i)

When the road becomes wet the equation becomes

$\frac{V}{2} = \sqrt{\mu' rg}$ ..... (ii)

Dividing equation (i) with equation (ii),

$$\frac{V}{\frac{V}{2}} = \frac{\sqrt{\mu rg}}{\sqrt{\mu' rg}}$$

$$\therefore \quad 2 = \frac{\sqrt{\mu}}{\sqrt{\mu'}}$$

$$\therefore \quad \mu' = \frac{\mu}{4}$$

Question 110

A wire of length 3 m connected in the left gap of a meter-bridge balances 8Ω resistance in the right gap at a point, which divides the bridge wire in the ratio $3:2$. The length of the wire corresponding to resistance of 1Ω is

Options:

A. 1 m

B. 0.75 m

C. 0.5 m

D. 0.25 m

Answer: D

Solution:

Let R_1 be the resistance of 3 m long wire connected in the left gap.

For meter-bridge, $\frac{R_1}{R_2} = \frac{l_1}{l_2}$

$$\therefore \quad \frac{R_1}{8} = \frac{3}{2}$$
$$\therefore \quad R_1 = \frac{3}{2} \times 8 = 12\Omega$$

Length of the wire corresponding to the resistance of 1Ω is $l = \frac{3}{12} = 0.25$ m

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Question 111

By adding soluble impurity in a liquid, angle of contact

Options:

A. decreases

B. increases

C. remains unchanged

D. first increases and then decreases

Answer: A

Solution:

The angle of contact, also known as the contact angle, is the angle at which a liquid/vapor interface meets the solid surface. It quantifies the wettability of a solid surface by a liquid via the Young equation:

$\cos(\theta) = \frac{\sigma_{SG} - \sigma_{SL}}{\sigma_{LG}}$

where:

- θ is the contact angle
- σ_{SG} is the interfacial tension between the solid and gas
- σ_{SL} is the interfacial tension between the solid and liquid
- σ_{LG} is the interfacial tension between the liquid and gas

When a soluble impurity is added to a liquid, it affects the surface tension of the liquid-gas interface (σ_{LG}). If the solute has a surface-active property, meaning it tends to accumulate at the surface, it typically reduces the surface tension of the liquid.

A lower σ_{LG} (surface tension of the liquid) influences the balance of forces and consequently could decrease the contact angle if the impurity does not significantly affect the solid-liquid and solid-gas interfacial tensions. A decreased contact angle means the liquid spreads out more on the surface, leading to better wetting.

So, when considering the effects of a soluble impurity in a liquid, and assuming the impurity is surface-active and does not significantly affect the solid-liquid and solid-gas interfacial tensions, the correct answer is:

Option A: decreases

This is because the addition of the impurity decreases the surface tension of the liquid, typically resulting in a smaller contact angle and increased wetting.

Question 112

For a common emitter configuration, if 'α' and 'β' have their usual meanings, the incorrect relation between 'α' and 'β' is

Options:

A. $\frac{1}{\alpha} = \frac{1}{\beta} + 1$

B. $\alpha = \frac{\beta}{1-\beta}$

C. $\alpha = \frac{\beta}{1+\beta}$

D. $\frac{1}{\beta} = \frac{1}{\alpha} - 1$

Answer: B

Solution:

In a common emitter configuration, the parameters 'α' (alpha) and 'β' (beta) are related as follows:

α is the current gain in a common base configuration and β is the current gain in a common emitter configuration. The relationships between 'α' and 'β' are given by:

$\beta = \frac{\alpha}{1-\alpha}$

and

$\alpha = \frac{\beta}{\beta+1}$

Let's examine the given options one by one to determine the incorrect relation:

Option A: $\frac{1}{\alpha} = \frac{1}{\beta} + 1$

Rewriting this equation:

$\frac{1}{\alpha} = \frac{1}{\beta} + 1$

This can be written as:

$\frac{1}{\alpha} = \frac{1+\beta}{\beta}$

Therefore, this is a correct relation.

Option B: $\alpha = \frac{\beta}{1-\beta}$

We know that:

$\alpha = \frac{\beta}{\beta+1}$

Clearly, $\frac{\beta}{1-\beta}$ does not match this, so this is the incorrect relation.

Option C: $\alpha = \frac{\beta}{1+\beta}$

We know that:

$\alpha = \frac{\beta}{\beta+1}$

This is the correct relation.

Option D: $\frac{1}{\beta} = \frac{1}{\alpha} - 1$

Rewriting this equation:

$\frac{1}{\beta} = \frac{1}{\alpha} - 1$

This can be written as:

$\frac{1}{\beta} = \frac{1-\alpha}{\alpha}$

From the relation $\beta = \frac{\alpha}{1-\alpha}$, inverting both sides we get:

$\frac{1}{\beta} = \frac{1-\alpha}{\alpha}$

Therefore, this is a correct relation.

In conclusion, the incorrect relation between 'α' and 'β' is Option B: $\alpha = \frac{\beta}{1-\beta}$.

Question 113

A simple pendulum of length 'l' and a bob of mass 'm' is executing S.H.M. of small amplitude 'A'. The maximum tension in the string will be (g $=$ acceleration due to gravity)

Options:

A. 2 mg

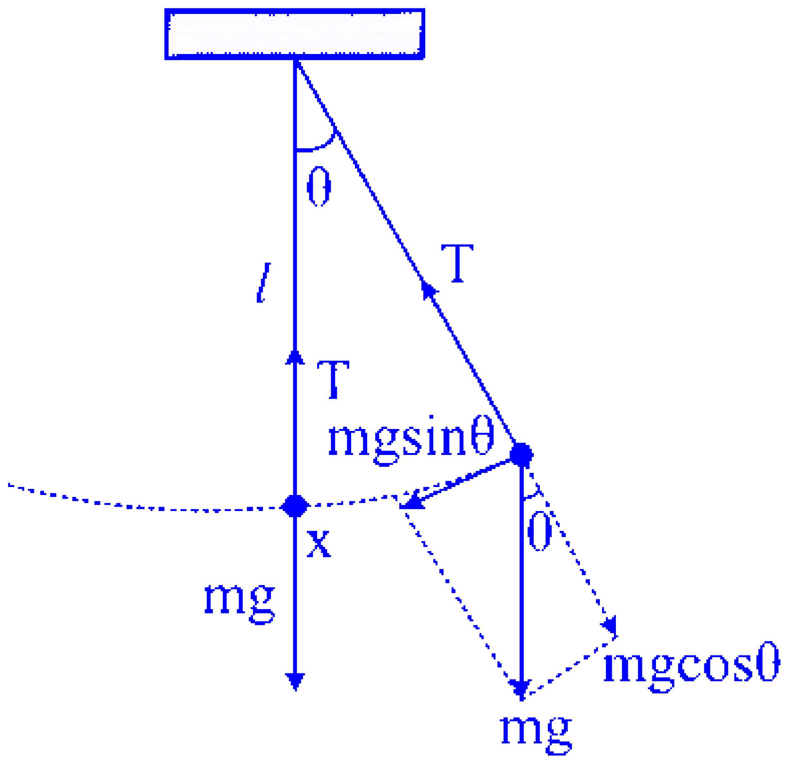
B. $\mathrm{mg}\left[1+\left(\frac{\mathrm{A}}{\ell}\right)^2\right]$

C. $\mathrm{mg}\left[1+\left(\frac{\mathrm{A}}{\ell}\right)\right]^2$

D. $mg\left[1+\left(\frac{\mathrm{A}}{\ell}\right)\right]$

Answer: B

Solution:



Tension in the string is given as,

$$\mathrm{T}' = \mathrm{mg}\cos\theta + \frac{\mathrm{mv}^2}{l}$$
$$\mathrm{T}'_{\max} = \mathrm{mg} + \frac{\mathrm{mv}^2}{l}$$

Now, time period $T = 2\pi\sqrt{\frac{l}{g}}$

$\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{l}}$

Maximum Velocity is given as $v_{max} = A\omega$

Substituting the values

$$T'_{max} = mg + \frac{m(A\omega)^2}{l}$$
$$= mg + \frac{\frac{mA^2 g}{l}}{l}$$
$$= mg + \frac{mA^2 g}{l^2}$$
$$= mg\left(1 + \left(\frac{A}{l}\right)^2\right)$$

Question 114

Which of the following combination of 7 identical capacitors each of $2\mu F$ gives a capacitance of $\frac{10}{11}\mu F$?

Options:

A. 5 in parallel and 2 in series

B. 4 in parallel and 3 in series

C. 3 in parallel and 4 in series

D. 2 in parallel and 5 in series

Answer: A

Solution:

For n identical capacitors connected in series, the equivalent capacitance is, $C_s = \frac{C}{n}$

Similarly, for m identical capacitors connected parallel to each other, the equivalent capacitance is, $C_p = mC$

Assuming the two combinations are connected in series, the net capacitance,

$$\frac{1}{C_{net}} = \frac{1}{mC} + \frac{n}{C} = \frac{11}{10}\mu F \quad \ldots.\left(\because C_{net} = \frac{10}{11}\mu F\right)$$

$\therefore$ for $C = 2\mu F$,

$$\frac{11 \times 2}{10} = \frac{1}{m} + n$$

$$\therefore \quad \frac{1}{m} + n = \frac{11}{5} \quad \ldots\ldots \text{(i)}$$

Substituting the values for m and n in equation (i) from each option, the correct answer can be found to be (A).

Question 115

The potential energy of a molecule on the surface of a liquid compared to the molecules inside the liquid is

Options:

A. zero

B. less

C. same

D. large

Answer: D

Solution:

The potential energy of a molecule on the surface of a liquid is higher compared to the molecules inside the liquid. This is because molecules on the surface are not surrounded by as many neighboring molecules as those inside, meaning they experience fewer attractive forces. Hence, the correct answer is :

Option D : large

Question 116

A progressive wave is given by, $Y = 12\sin(5t - 4x)$. On this wave, how far away are the two points having a phase difference of 90° ?

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{16}$

D. $\frac{\pi}{32}$

Answer: B

Solution:

Given, $y = 12\sin(5t - 4x)$

$$\therefore \quad y = 12\sin 2\pi\left(\frac{5t}{2\pi} - \frac{4x}{2\pi}\right)$$

Comparing above eq. with, $y = A\sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$

We get, $\lambda = \frac{2\pi}{4}$

Relation between phase difference and path difference is

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta \mathrm{x}$$

$$\therefore \quad \frac{\pi}{2} = \frac{2\pi}{\left(\frac{2\pi}{4}\right)}\Delta \mathrm{x}$$

$$\therefore \quad \Delta \mathrm{x} = \frac{\pi}{8}$$

Question 117

The de-Broglie wavelength (λ) of a particle is related to its kinetic energy (E) as

Options:

A. $\lambda \propto \mathrm{E}$

B. $\lambda \propto \mathrm{E}^{-1}$

C. $\lambda \propto \mathrm{E}^{\frac{1}{2}}$

D. $\lambda \propto E^{-\frac{1}{2}}$

Answer: D

Solution:

De-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mE}}$

$\therefore \quad \lambda \propto E^{-\frac{1}{2}}$

Question 118

For a purely inductive or a purely capacitive circuit, the power factor is

Options:

A. zero

B. 0.5

C. 1

D. ∞

Answer: A

Solution:

For a purely inductive or a purely capacitive circuit, the power factor is indeed zero. That makes Option A the correct choice. Let's explain why this is the case.

In AC circuits, the power factor is defined as the cosine of the phase angle (denoted as $\cos(\phi)$) between the voltage and the current. The power factor can range from -1 (which would be for a purely capacitive load with current leading the voltage by 90 degrees) to +1 (which would be for a purely resistive load with voltage and current in phase). If the power factor is zero, this means that the phase angle between the voltage and current is 90 degrees.

In a purely inductive circuit, the inductor causes the current to lag behind the voltage by 90 degrees, while in a purely capacitive circuit, the capacitor causes the current to lead the voltage by 90 degrees. In both cases, the current is out of phase with the voltage by 90 degrees, which gives us a phase angle $\phi = \pm 90^{\circ}$. The cosine of +90 degrees or -90 degrees is zero:

$\cos(90^{\circ}) = \cos(-90^{\circ}) = 0.$

This means that in both cases (pure inductance or pure capacitance), there is no real power being consumed or dissipated in the circuit. Instead, power is just oscillating back and forth between the source and the reactive component (the inductor or capacitor). This oscillating power is known as reactive power, and while it is transferred to and from the source, it is not converted into heat or work. Therefore, the real power (the part of power which does work or generates heat) in a purely inductive or capacitive circuit is zero, and hence the power factor is zero.

Question 119

The electric field intensity on the surface of a solid charged sphere of radius 'r' and volume charge density 'ρ' is ($\varepsilon_0 =$ permittivity of free space)

Options:

A. $\frac{\rho r}{3\varepsilon_0}$

B. $\frac{\rho}{4\pi\varepsilon_0 \mathrm{r}}$

C. zero

D. $\frac{5\rho \mathrm{r}}{6\varepsilon_0}$

Answer: A

Solution:

According to Gauss' theorem,

$\phi = \mathrm{E} \cdot \mathrm{A} = \frac{\mathrm{q}_{\mathrm{enc}}}{\varepsilon_0}$

But, $\mathrm{q}_{\mathrm{enc}} = \rho \times$ volume

$\therefore \quad \mathrm{q}_{\mathrm{enc}} = \rho\left(\frac{4}{3}\pi \mathrm{r}^3\right)$

And area is $A = 4\pi r^2$

$$\therefore \quad E\left(4\pi r^2\right) = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_0}$$

$$\therefore \quad E = \frac{\rho r}{3\varepsilon_0}$$

Question 120

A body is said to be opaque to the radiation if (a, r and t are coefficient of absorption, reflection and transmission respectively)

Options:

A. $\mathrm{t}=0$ and $\mathrm{a}+\mathrm{r}=1$

B. $\mathrm{a}=\mathrm{r}=\mathrm{t}$

C. $t \neq 0$

D. $\mathrm{a}=0, \mathrm{r}=1, \mathrm{t}=1$

Answer: A

Solution:

Opaque body does not transmit any radiation directly,

$\therefore \quad t=0$
$\therefore \quad a+r=1$

Question 121

In a thermodynamic system, ΔU represents the increases in its internal energy and dW is the work done by the system then correct statement out of the following is

Options:

A. $\Delta \mathrm{U}=\mathrm{dW}$ is an isothermal process

B. $\Delta \mathrm{U}=-\mathrm{dW}$ is an adiabatic process

C. $\Delta \mathrm{U}=-\mathrm{dW}$ is an isothermal process

D. $\Delta \mathrm{U}=\mathrm{dW}$ is an adiabatic process

Answer: B

Solution:

For an isothermal process, $\Delta U = 0$. According to first law of thermodynamics,

$\Delta Q = \Delta U + dW$

For an adiabatic process,

$$\begin{aligned} & \Delta Q = 0 \\ \therefore \quad & \Delta U = -dW \end{aligned}$$

$\therefore$ It is an adiabatic process.

Question 122

A combination of two thin lenses in contact have power $+10\mathrm{D}$. The power reduces to $+6\mathrm{D}$ when the lenses are $0.25\ \mathrm{m}$ apart. The power of individual lens is

Options:

A. $5\mathrm{D}, 5\mathrm{D}$

B. $6\mathrm{D}, 4\mathrm{D}$

C. $7\mathrm{D}, 3\mathrm{D}$

D. $8\mathrm{D}, 2\mathrm{D}$

Answer: D

Solution:

When in contact, the equivalent power of two thin lenses is,

$P_1 + P_2 = 10$ ..... (i)

When the lenses are separated by 0.25 m,

$P_1 + P_2 - 0.25 P_1 P_2 = 6$

$\therefore \quad 0.25P_1P_2 = 4$

$\therefore \quad P_1P_2 = 16$

$$P_1 - P_2 = \sqrt{(P_1 + P_2)^2 + 4P_1P_2}$$

$\therefore \quad P_1 - P_2 = \sqrt{36} = 6 \quad ..... \text{(ii)}$

Adding equations (i) and (ii), we get,

$P_1 = 8D$

$\therefore \quad P_2 = 10 - 8 = 2D$

Question 123

The reciprocal of the total effective resistance of LCR a.c. circuit is called

Options:

A. impedance

B. admittance

C. resistance

D. inductive and capacitive reactance

Answer: B

Solution:

The total effective resistance of an LCR (inductor-capacitor-resistor) AC circuit is known as impedance, which is a complex quantity comprising the actual resistance and the reactances due to the inductance and capacitance. However, the reciprocal of impedance is called admittance. Admittance is a measure of how easily a circuit will allow electrical current to flow.

In mathematical terms, if the impedance Z is a complex number represented as :

$Z = R + jX$

where R is the resistance, X is the reactance, and j is the imaginary unit.

The admittance Y is the reciprocal of the impedance :

$Y = \frac{1}{Z}$

So the correct answer to the question is :

Option B : admittance

Question 124

If the radius of the first Bohr orbit is 'r' then the de-Broglie wavelength of the electron in the 4^{th} orbit will be

Options:

A. $4\pi r$

B. $6\pi r$

C. $8\pi r$

D. $\frac{\pi r}{4}$

Answer: C

Solution:

According to Bohr's second postulate,

$\frac{nh}{2\pi} = mvr_n$

$\therefore$ de-Broglie wavelength, $\lambda_n = \frac{h}{mv} = \frac{2\pi r_n}{n}$

Also, $r_n \propto n^2$

$\therefore$ The de-Broglie wavelength of the electron in the 4^{th} orbit is:

$$\lambda_4 = \frac{2\pi r_4}{4} = \frac{2\pi \times (16r)}{4}$$
$$\therefore \quad \lambda_4 = 8\pi r$$

Question 125

A string of length 'L' fixed at one end carries a body of mass 'm' at the other end. The mass is revolved in a circle in the horizontal plane about a vertical axis passing through the fixed end of the string. The

string makes angle 'θ' with the vertical. The angular frequency of the body is 'ω'. The tension in the string is

Options:

A. $mL^2\omega$

B. $mL\omega^2$

C. $\frac{\omega^2}{mL}$

D. $\frac{m\omega^2}{L}$

Answer: B

Solution:

In case of conical pendulum, the tension in the string provides the necessary centripetal force.

$\therefore \quad T = mr\omega^2 = mL\omega^2 \quad$ .... (Here, r = L)

Question 126

The temperature of a gas is $-68°C$. To what temperature should it be heated, so that the r.m.s. velocity of the molecules be doubled?

Options:

A. $357°C$

B. $457°C$

C. $547°C$

D. $820°C$

Answer: C

Solution:

$T_1 = -68°C = -68 + 273\ K = 205\ K$

R.M.S. velocity, $v_{rms} \propto \sqrt{T}$

$$\frac{(v_{ms})_2}{(v_{ms})_1} = \sqrt{\frac{T_2}{T_1}} = 2$$

$\therefore \quad \frac{T_2}{T_1} = 4$

$\therefore \quad T_2 = 4 \times 205 = 820\ K$

$\therefore \quad T_2 = 547°C$

Question 127

The displacement of a particle executing S.H.M. is $x = a\sin(\omega t - \phi)$. Velocity of the particle at time $t = \frac{\phi}{\omega}$ is $(\cos 0° = 1)$

Options:

A. $\omega \cos \phi$

B. $a\omega$

C. $\omega \cos 2\phi$

D. $-a\omega \cos 2\phi$

Answer: B

Solution:

To calculate the velocity of a particle in simple harmonic motion (S.H.M.), we use the derivative of the displacement function with respect to time. The displacement function given is:

$x = a\sin(\omega t - \phi)$

The velocity v is the first derivative of displacement x with time t:

$v = \frac{dx}{dt}$

Let's compute the derivative:

$v = \frac{d}{dt}[a\sin(\omega t - \phi)]\ v = a\cos(\omega t - \phi) \cdot \frac{d}{dt}(\omega t - \phi)\ v = a\cos(\omega t - \phi) \cdot \omega\ v = a\omega\cos(\omega t - \phi)$

We need to calculate this velocity at time $t = \frac{\phi}{\omega}$:

$v = a\omega\cos\left(\omega \cdot \frac{\phi}{\omega} - \phi\right)\ v = a\omega\cos(\phi - \phi)\ v = a\omega\cos(0°)\ v = a\omega \cdot 1\ v = a\omega$

Therefore, the correct answer is Option B:

$a\omega$

Question 128

A uniformly charged semicircular arc of radius 'r' has linear charge density 'λ'. The electric field at its centre is (ε_0 = permittivity of free space)

Options:

A. $\frac{\lambda}{4\varepsilon_0}$

B. $\frac{2\varepsilon_0}{\lambda}$

C. $\frac{\lambda}{4\varepsilon_0 \mathrm{r}}$

D. $\frac{2\pi\varepsilon_0}{\lambda}$

Answer: C

Solution:

$$\lambda = \frac{\mathrm{q}}{l}$$

$$\therefore \quad \mathrm{q} = \lambda \times l = \lambda \times \pi\mathrm{r}$$

$\therefore$ The electric field at its centre is,

$$\mathrm{E} = \frac{\mathrm{q}}{4\pi\varepsilon_0\mathrm{r}^2} = \frac{\lambda\pi\mathrm{r}}{4\pi\varepsilon_0\mathrm{r}^2}$$

$$\therefore \quad \mathrm{E} = \frac{\lambda}{4\varepsilon_0\mathrm{r}}$$

Question 129

A sphere, a cube and a thin circular plate all made of same material and having the same mass are heated to same temperature of $200^\circ\mathrm{C}$. When these are left in a room.

Options:

A. the sphere reaches room temperature fast

B. the cube reaches room temperature fast

C. the circular plate reaches room temperature fast

D. all will reach the room temperature simultaneously

Answer: C

Solution:

$\rho = \frac{M}{V}$;

Given: three objects are of same materials ρ is same. Also, $\mathrm{M} = \text{same}$,

$\therefore$ volume of objects is same.

For constant volume, amongst the given objects surface area of the plate is maximum.

Hence according to Stefan's law $\frac{\mathrm{dQ}}{\mathrm{dt}} \propto \mathrm{AT}^4$

As A_{plate} is maximum, the plate will cool fastest and reaches room temperature fast.

Question 130

For emission of light, a light emitting diode (LED) is

Options:

A. always used in reverse biased condition

B. never used in forward or reserve biased condition

C. used both in forward and reverse biased condition depending upon its application

D. always used in forward biased condition

Answer: D

Solution:

For a light-emitting diode (LED) to emit light, it must be used in a forward-biased condition. Therefore, the correct answer is:

Option D : always used in forward biased condition

When an LED is forward biased, the electrons in the n-type material have enough energy to cross the p-n junction and recombine with holes in the p-type material. This recombination process results in the release of energy in the form of photons, which is the light that we see emitted from the LED. On the other hand, when an LED is in reverse bias, it does not emit light because the potential barrier at the p-n junction is increased, preventing electron-hole recombination.

Question 131

A solenoid of length $0.4 \mathrm{~m}$ and having 500 turns of wire carries a current $3 \mathrm{~A}$. A thin coil having 10 turns of wire and radius $0.1 \mathrm{~m}$ carries current $0.4 \mathrm{~A}$. the torque required to hold the coil in the middle of the solenoid with its axis perpendicular to the axis of the solenoid is $\left(\mu_0=4 \pi \times 10^{-7}\right.$ SI units, $\left.\pi^2=10\right)\left(\sin 90^{\circ}=1\right)$

Options:

A. $3 \times 10^{-6} \mathrm{~Nm}$

B. $12 \times 10^{-6} \mathrm{~Nm}$

C. $6 \times 10^{-4} \mathrm{~Nm}$

D. $24 \times 10^{-6} \mathrm{~Nm}$

Answer: C

Solution:

The equation for torque is given by:

$\tau=\mathrm{NIAB} \sin \theta$

As axis of coil and solenoid are perpendicular to each other, $\theta=90^{\circ}$ and $\sin \theta=1$

$$
\begin{aligned}
\therefore \quad \tau & =\mu_0 \mathrm{nI}_1 \times \text { NIA } \\
& =4 \pi \times 10^{-7} \times \frac{500}{0.4} \times 3 \times 10 \times 0.4 \times \pi \times 10^{-2} \\
\tau & =6 \times 10^{-4} \mathrm{Nm}
\end{aligned}
$$

Question 132

In semiconductors at room temperature,

Options:

A. the valence band is completely filled

B. the conduction band is completely filled

C. the condition band is partially filled and valence band is partially empty

D. the valence band is completely filled and the conduction band is partially filled

Answer: A

Solution:

In semiconductors at room temperature, the behavior of electrons across the valence and conduction bands is crucial for understanding their electrical properties. Let's clarify the options given and identify the correct answer:

- **Option A: the valence band is completely filled**

At absolute zero temperature, the valence band is indeed completely filled with electrons, and the conduction band is empty. However, at room temperature, thermal energy excites some electrons from the valence band to the conduction band, creating holes (absence of electrons) in the valence band. Thus, while the valence band may still be considered "full" in a broader sense, it's more accurate to say it contains holes due to the excitation of electrons.

- **Option B: the conduction band is completely filled**

This is incorrect for semiconductors at room temperature. The conduction band becomes partially filled due to electron excitation from the valence band, but it is not completely filled under normal conditions.

- **Option C: the condition band is partially filled and valence band is partially empty**

This statement introduces a misunderstanding. While the concept of the conduction band being partially filled is correct, saying the valence band is "partially empty" might be misleading. The valence band is less than fully occupied due to the presence of holes, but it's a result of electron movement, not an inherent emptiness.

- **Option D: the valence band is completely filled and the conduction band is partially filled**

This option might seem correct at a glance, but it misrepresents the state of the valence band at room temperature. The valence band has holes (unoccupied states) due to electrons being excited to the conduction band, which contradicts the notion of being "completely filled."

Correct Understanding

At room temperature, semiconductors have their valence band almost filled with some electrons excited to the conduction band, creating holes in the valence band. The conduction band is partially filled with these excited electrons. This scenario is crucial for the semiconductor's ability to conduct electricity, as both the excited electrons in the conduction band and the holes left behind in the valence band contribute to electrical conduction.

Given the explanations, the precise wording should reflect that the valence band has electrons but also contains holes due to thermal excitation, and the conduction band is partially filled with these excited electrons. The options provided do not perfectly capture this nuanced state. However, understanding semiconductors involves recognizing the excitation of electrons from the valence to the conduction band, leaving the valence band less than fully occupied (due to holes) and the conduction band partially occupied by electrons at room temperature.

Question 133

Considering earth to be a sphere of radius 'R' having uniform density 'ρ', then value of acceleration due to gravity 'g' in terms of R, ρ and G is

Options:

A. $g=\sqrt{\frac{3\pi R}{\rho G}}$

B. $\mathrm{g}=\sqrt{\frac{4}{3}\pi\rho\mathrm{GR}}$

C. $\mathrm{g}=\frac{4}{3}\pi\rho\mathrm{GR}$

D. $g=\frac{GM}{\rho R^2}$

Answer: C

Solution:

To derive an expression for the acceleration due to gravity 'g' at the surface of the Earth, in terms of its radius 'R', its uniform density 'ρ', and the gravitational constant 'G', we can start by calculating the mass 'M' of the Earth in terms of its density and volume.

The volume 'V' of a sphere is given by

$V=\frac{4}{3}\pi R^3$

For a sphere with uniform density 'ρ', the mass 'M' can be calculated by multiplying the volume by the density:

$M = \rho V = \rho\left(\frac{4}{3}\pi R^3\right)$

Now that we have the mass, we can use Newton's law of universal gravitation to find the force 'F' exerted on a mass 'm' at the surface of the Earth:

$F = \frac{GMm}{R^2}$

The acceleration 'g' due to gravity at the Earth's surface is simply the force per unit mass 'm':

$g = \frac{F}{m} = \frac{GM}{R^2}$

Substituting the expression for 'M' in terms of 'ρ' and 'R', we get:

$g = \frac{G\left(\rho\left(\frac{4}{3}\pi R^3\right)\right)}{R^2}$

When we simplify this expression, we have:

$g = \frac{G\rho\frac{4}{3}\pi R^3}{R^2}\ g = \frac{4}{3}\pi G\rho R$

Therefore, the correct answer is:

Option C

$g = \frac{4}{3}\pi\rho GR$

Question 134

The equation of the wave is $\mathrm{Y} = 10\sin\left(\frac{2\pi \mathrm{t}}{30} + \alpha\right)$ If the displacement is $5\ \mathrm{cm}$ at $\mathrm{t} = 0$ then the total phase at $\mathrm{t} = 7.5\ \mathrm{s}$ will be $(\sin 30^\circ = 0.5)$

Options:

A. $\frac{\pi}{3}$ rad

B. $\frac{\pi}{2}$ rad

C. $\frac{2\pi}{5}$ rad

D. $\frac{2\pi}{3}$ rad

Answer: D

Solution:

The given wave equation is:

$Y = 10\sin\left(\frac{2\pi t}{30} + \alpha\right)$

At t = 0 , the displacement Y is given as 5 cm. So, let's find the phase α using this information :

$5 = 10\sin(\alpha)$

$\frac{5}{10} = \frac{1}{2} = \sin(\alpha)$

Since $\sin(30^\circ) = \frac{1}{2}$, we can say that $\alpha = 30^\circ$ or $\alpha = \frac{\pi}{6}$ radians, considering that sine is positive in the first and second quadrants, and seeing that α represents a phase shift, it usually takes the smallest positive angle that satisfies the equation.

Now, let's calculate the total phase at $t = 7.5$ s :

$\text{Total phase} = \frac{2\pi t}{30} + \alpha$

Plug in the values:

$\text{Total phase} = \frac{2\pi \cdot 7.5}{30} + \frac{\pi}{6}$

$\text{Total phase} = \frac{2\pi \cdot 1}{4} + \frac{\pi}{6} = \frac{\pi}{2} + \frac{\pi}{6}$

To add these fractions, we first find a common denominator :

$\text{Total phase} = \frac{3\pi}{6} + \frac{\pi}{6} = \frac{4\pi}{6}$

Now we simplify the fraction :

$\text{Total phase} = \frac{2\pi}{3}$

Therefore, the total phase at $t = 7.5$ s is $\frac{2\pi}{3}$ radians, which corresponds to Option D.

Question 135

The bob of simple pendulum of length 'L' is released from a position of small angular displacement θ. Its linear displacement at time 't' is ($g =$ acceleration due to gravity)

Options:

A. $L\theta \cos\left[\sqrt{\frac{g}{L}} \cdot t\right]$

B. $L\theta \sin\left[2\pi\sqrt{\frac{g}{L}} \cdot t\right]$

C. $L\theta \cos\left[2\pi\sqrt{\frac{g}{L}} \cdot t\right]$

D. $L\theta \sin\left[\sqrt{\frac{g}{L}} \cdot t\right]$

Answer: A

Solution:

Equation for displacement for a particle performing S.H.M. is,

$y = A \cos \omega t = L \cos\left(\frac{2\pi}{T} \times t\right)$

But time period of a simple pendulum,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\therefore \quad y = L \cos\left(\frac{2\pi}{2\pi\sqrt{\frac{L}{g}}} \times t\right) = L \cos\left(\sqrt{\frac{g}{L}} \times t\right)$$

$\therefore$ The linear displacement is,

$s = y\theta = L\theta \cos\left[\sqrt{\frac{g}{L}} \times t\right]$

Question 136

The value of acceleration due to gravity at a depth 'd' from the surface of earth and at an altitude 'h' from the surface of earth are in the ratio

Options:

A. $1 : 1$

B. $\frac{R-2h}{R-d}$

C. $\frac{R-d}{R-2h}$

D. $\frac{\mathrm{R-d}}{\mathrm{R-h}}$

Answer: C

Solution:

$$\frac{g_d}{g_h}=\frac{g\left[1-\frac{d}{R}\right]}{g\left[1-\frac{2h}{R}\right]}$$
$$\frac{g_d}{g_h}=\frac{R-d}{R-2h}$$

Question 137

A magnetic field of 2×10^{-2} T acts at right angles to a coil of area $100\ \mathrm{cm}^2$ with 50 turns. The average e.m.f. induced in the coil is 0.1 V, when it is removed from the field in time 't'. The value of 't' is

Options:

A. 2×10^{-3} s

B. 0.5 s

C. 0.1 s

D. 1 s

Answer: C

Solution:

$$\mathrm{e}=-\frac{\mathrm{d}\phi}{\mathrm{dt}}=-\frac{(\phi_2-\phi_1)}{\mathrm{t}}=-\frac{(0-\mathrm{NBA})}{\mathrm{t}}$$

$$\therefore\quad 0.1=\frac{50\times2\times10^{-2}\times10^{-2}}{\mathrm{t}}$$

$\therefore$ The value of 't' is,

$$\mathrm{t}=\frac{10^{-2}}{0.1}=0.1\ \mathrm{s}$$

Question 138

In Young's double slit experiment, 8^{th} maximum with wavelength 'λ_1' is at a distance 'd_1' from the central maximum and 6^{th} maximum with wavelength 'λ_2' is at a distance 'd_2'. Then $\frac{d_2}{d_1}$ is

Options:

A. $\frac{3\lambda_1}{4\lambda_2}$

B. $\frac{3\lambda_2}{4\lambda_1}$

C. $\frac{4\lambda_1}{3\lambda_2}$

D. $\frac{4\lambda_2}{3\lambda_1}$

Answer: B

Solution:

$$d \propto n\lambda$$

$$\therefore \quad \frac{d_2}{d_1} = \frac{n_2\lambda_2}{n_1\lambda_1} = \frac{6\lambda_2}{8\lambda_1}$$

$$\therefore \quad \frac{d_2}{d_1} = \frac{3\lambda_2}{4\lambda_1}$$

Question 139

The alternating e.m.f. induced in the secondary coil of a transformer is mainly due to

Options:

A. varying electric field

B. varying magnetic field

C. the iron core

D. heat produced in the coil

Answer: B

Solution:

The secondary coil experiences an emf as a result of the changing magnetic field. Therefore, a changing magnetic field is primarily responsible for the transformer voltage that is induced in a transformer's secondary coil.

Question 140

The efficiency of a heat engine is 'η' and the coefficient of performance of a refrigerator is 'β'. Then

Options:

A. $\eta = \frac{1}{\beta}$

B. $\eta = \frac{1}{\beta+1}$

C. $\eta\beta = \frac{1}{2}$

D. $\eta = \frac{1}{\beta-1}$

Answer: B

Solution:

$$\beta = \frac{T_2}{T_1 - T_2}$$

$$\eta = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1}$$

$$\therefore \quad \eta = \frac{1}{1 + \frac{T_2}{T_1 - T_2}}$$

$$\therefore \quad \eta = \frac{1}{1 + \beta}$$

Question 141

A conducting sphere of radius $0.1 \mathrm{~m}$ has uniform charge density $1.8 \mu \mathrm{C} / \mathrm{m}^2$ on its surface. The electric field in free space at radial distance $0.2 \mathrm{~m}$ from a point on the surface is ($\varepsilon_0=$ permittivity of free space)

Options:

A. $\frac{6 \times 10^{-6}}{\varepsilon_0} \mathrm{Vm}^{-1}$

B. $\frac{6 \times 10^{-8}}{\varepsilon_0} \mathrm{Vm}^{-1}$

C. $\frac{2 \times 10^{-7}}{\varepsilon_0} \mathrm{Vm}^{-1}$

D. $\frac{1 \times 10^{-7}}{\varepsilon_0} \mathrm{Vm}^{-1}$

Answer: C

Solution:

To find the electric field at a distance of $0.2 \mathrm{~m}$ from the surface of the conducting sphere, we can use Gauss's Law. For a spherical charge distribution, the electric field at an external point is the same as if all the charge were concentrated at the center of the sphere.

First, we calculate the total charge Q on the sphere. The charge density σ is the charge per unit area, so multiplying it by the surface area A gives us the total charge Q.

The surface area of a sphere is given by:

$A=4 \pi r^2$

Where r is the radius of the sphere. Plugging in the given radius of $0.1 \mathrm{~m}$,

$A=4 \pi(0.1 \mathrm{~m})^2=0.04 \pi \mathrm{~m}^2$

The charge density σ is given as $1.8 \mu \mathrm{C} / \mathrm{m}^2$, which is $1.8 \times 10^{-6} \mathrm{C} / \mathrm{m}^2$. Now compute the total charge Q:

$Q=\sigma \cdot A=\left(1.8 \times 10^{-6} \mathrm{C} / \mathrm{m}^2\right)\left(0.04 \pi \mathrm{~m}^2\right)$

$Q=\left(1.8 \times 10^{-6}\right) \cdot(0.04 \pi) \mathrm{C}=7.2 \times 10^{-8} \pi \mathrm{C}$

Gauss's Law states that the electric field E multiplied by the surface area of an imaginary sphere $A^{\prime}$ that encloses the charge is equal to the total charge Q enclosed divided by the permittivity ε_0:

$E \cdot A' = \frac{Q}{\varepsilon_0}$

For a radial distance of 0.2 m from the surface, the actual radial distance from the center of the sphere will be the sum of the sphere's radius (0.1 m) and the distance from the surface (0.2 m), giving us:

$R = 0.1 \text{ m} + 0.2 \text{ m} = 0.3 \text{ m}$

The area of the Gaussian surface A' is now:

$A' = 4\pi(0.3 \text{ m})^2 = 0.36\pi \text{ m}^2$

Plugging the values back into the equation we have:

$E = \frac{7.2\times10^{-8}\pi \text{ C}}{\varepsilon_0 \cdot 0.36\pi \text{ m}^2}$

$E = \frac{7.2\times10^{-8}}{0.36\varepsilon_0}$

$E = \frac{7.2}{0.36} \times \frac{10^{-8}}{\varepsilon_0}$

$E = 20 \times \frac{10^{-8}}{\varepsilon_0} \text{Vm}^{-1}$

$E = 2 \times \frac{10^{-7}}{\varepsilon_0} \text{Vm}^{-1}$

Now, the value obtained is that the electric field E at a radial distance of 0.2 m from the sphere's surface is equal to:

$E = \frac{2\times10^{-7}}{\varepsilon_0} \text{Vm}^{-1}$

This matches Option C.

Question 142

If I_0 is the intensity of the principal maximum in the single slit diffraction pattern, then what will be the intensity when the slit width is doubled?

Options:

A. $\frac{I_0}{2}$

B. I_0

C. $4I_0$

D. $2I_0$

Answer: B

Solution:

The slit width has no effect on the intensity of the principal maximum in the single-slit diffraction pattern.

$\therefore$ The intensity will be same, I_0.

Question 143

Two circular coils made from same wire but radius of 1^{st} coil is twice that of 2^{nd} coil. If magnetic field at their centres is same then ratio of potential difference applied across them is (1^{st} to 2^{nd} coil)

Options:

A. 2

B. 3

C. 4

D. 6

Answer: C

Solution:

Magnetic fields at the centre of the coils are equal.

$$\therefore \quad \frac{\mu_0 I_1}{2r_1} = \frac{\mu_0 I_2}{2r_2}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{r_1}{r_2} = \frac{2r}{r} \quad \ldots(\text{given } r_1 = 2r_2)$$

$$\therefore \quad \frac{I_1}{I_2} = 2 \quad \ldots.(i)$$

The resistance through the coil,

$R \propto l$

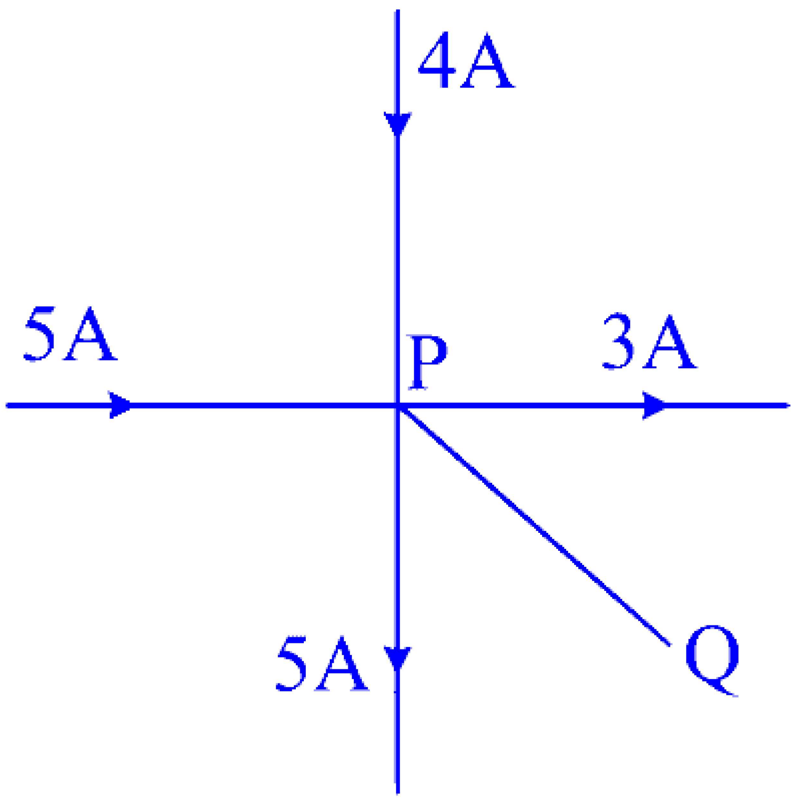
Here, $l = 2\pi r$

$$\therefore \quad \frac{R_1}{R_2} = \frac{2\pi r_1}{2\pi r_2} = 2 \quad .... \text{(ii)}$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{I_1 R_1}{I_2 R_2} = 2 \times 2 = 4 \quad ...\text{[From (i) and (ii)]}$$

Question 144

Five current carrying conductors meet at point P. What is the magnitude and direction of the current in conductor PQ?



Options:

A. 1 A form Q to P

B. 1 A form P to Q

C. 3 A form P to Q

D. 2 A form Q to P

Answer: B

Solution:

According to Kirchhoff's first law, at point P,

$5\text{ A}+4\text{ A}+\text{I}-5\text{ A}-3\text{ A}=0$

$\text{I}=-1\text{ A}$

$\therefore \quad I=-1A$

1 A current flows from P to Q.

Question 145

Water rises in a capillary tube of radius 'r' upto a height 'h'. The mass of water in a capillary is 'm'. The mass of water that will rise in a capillary tube of radius $\frac{'r'}{3}$ will be

Options:

A. 3 m

B. $\frac{\text{m}}{3}$

C. m

D. $\frac{2\text{ m}}{3}$

Answer: B

Solution:

$h \propto \frac{1}{r}$

$\therefore$ Mass of water in a capillary, $m=\pi r^2 h\rho$

$\therefore \quad m \propto r^2 h$

$\therefore \quad m \propto r^2/r$

$\therefore \quad \frac{m_2}{m}=\frac{\frac{r}{3}}{r}$

$\therefore \quad m_2=\frac{m}{3}$

Question 146

A person with machine gun can fire 50 g bullets with a velocity of $240 \mathrm{~m/s}$. A $60 \mathrm{~kg}$ tiger moves towards him with a velocity of $12 \mathrm{~m/s}$. In order to stop the tiger in track, the number of bullets the person fires towards the tiger is

Options:

A. 50

B. 60

C. 70

D. 80

Answer: B

Solution:

In order to stop the tiger, the momentum of the bullets fired must be equal to the momentum of the tiger.

$$\therefore \quad \mathrm{MV} = \mathrm{nmv}$$
$$\therefore \quad \mathrm{n} = \frac{\mathrm{MV}}{\mathrm{mv}} = \frac{60 \times 12}{50 \times 10^{-3} \times 240}$$
$$\therefore \quad \mathrm{n} = 60$$

Question 147

If 'l' is the length of the open pipe, 'r' is the internal radius of the pipe and 'V ' is the velocity of sound in air then fundamental frequency of open pipe is

Options:

A. $\frac{\mathrm{V}}{(l+0.3\mathrm{r})}$

B. $\frac{\mathrm{V}}{(l+1.2\mathrm{r})}$

C. $\frac{V}{(l+0.6r)}$

D. $\frac{V}{2(l+1.2r)}$

Answer: D

Solution:

For an open organ pipe, the length of the pipe with end correction is given as:

$$L = l + 2e = l + 2 \times 0.6r$$
$$L = l + 1.2r$$

$\therefore$ The fundamental frequency of open pipe is:

$$f = \frac{v}{2L}$$
$$f = \frac{v}{2(l + 1.2r)}$$

Question 148

The angle of deviation produced by a thin prism when placed in air is 'δ_1' and that when immersed in water is 'δ_2'. The refractive index of glass and water are $\frac{3}{2}$ and $\frac{4}{3}$ respectively. The ratio $\delta_1 : \delta_2$ is

Options:

A. 1 : 2

B. 2 : 1

C. 1 : 4

D. 4 : 1

Answer: D

Solution:

For thin prism, $\delta = (\mu - i)A$

Given, $\mu_1 = \frac{\mu_{\text{glass}}}{\mu_{\text{air}}} = \frac{3}{2}$ and $\frac{\mu_{\text{water}}}{\mu_{\text{air}}} = \frac{4}{3}$

$$\therefore \quad \mu_2 = \frac{\mu_{\text{glass}}}{\mu_{\text{water}}} = \frac{\frac{3}{2}}{\frac{2}{3}} = \frac{9}{8}$$

$$\therefore \quad \frac{\delta_1}{\delta_2} = \frac{\mu_1 - 1}{\mu_2 - 1} = \frac{\frac{3}{2} - 1}{\frac{9}{8} - 1}$$

$$\therefore \quad \frac{\delta_1}{\delta_2} = 4$$

Question 149

A thin uniform rod of mass 'm' and length 'P' is suspended from one end which can oscillate in a vertical plane about the point of intersection. It is pulled to one side and then released. It passes through the equilibrium position with angular speed 'ω'. The kinetic energy while passing through mean position is

Options:

A. $\mathrm{ml}^2\omega^2$

B. $\frac{\mathrm{m}l^2\omega^2}{4}$

C. $\frac{\mathrm{m}l^2\omega^2}{6}$

D. $\frac{\mathrm{m}l^2\omega^2}{12}$

Answer: C

Solution:

The kinetic energy of the rod while passing through the mean position will be,

$$\begin{aligned} \text{K.E.} &= \frac{1}{2}\mathrm{I}\omega^2 \\ &= \frac{1}{2}\frac{\mathrm{m}l^2}{3} \times \omega^2 = \frac{\mathrm{m}l^2\omega^2}{6} \end{aligned}$$

Question 150

Magnetic field at the centre of the hydrogen atom due to motion of electron in n^{th} orbit is proportional to

Options:

A. n^4

B. n^{-3}

C. n^3

D. n^{-5}

Answer: D

Solution:

The radius of the n^{th} Bohr orbit is, $r_n \propto n^2$.

The angular velocity of the electron, $\omega_{\mathrm{n}} \propto \frac{1}{\mathrm{n}^3}$

Also, current $I_n = \frac{q}{T_n} = \frac{q\omega_n}{2\pi}$

$\therefore \quad I_n \propto \omega_n \propto \frac{1}{n^3}$

Now, $B_n = \frac{\mu_0 I_n}{2r_n}$

$\therefore \quad B_n \propto \frac{1}{n^3} \times \frac{1}{n^2}$

$\therefore \quad B_n \propto \frac{1}{n^5}$